

# Ko and Dame Endgames under Area Scoring

by Robert Jasiek

Version 3

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## Preface

This document studies endgame positions that have basic endgame kos, can have dame, and can have ko threats.

The positions shall *not have teire*, any other remaining endgames, nor unusual aspects. If they had, strategy would have to be modified appropriately compared to the discussion here. In particular, positions with teire still demand basic research before they could be treated in general like the types of positions in this document. Subject to these restrictions, this document studies and classifies *all* possible positions that have basic endgame kos, can have dame, and can have ko threats.

In this document, ko captures, plays on dame, or passes are not called ko threats. Plays threatening more are called ko threats.

This document assumes usage of rules with the following characteristics: area scoring, positional superko (a play may not recreate any earlier position), and 2 ending passes. Other ko rules or numbers of passes for an ending could be used instead but, practically speaking, endgame strategy would be the same if area scoring is still used.

Positive scores favour Black. Negative scores favour White. E.g., the score -1 means that White gets 1 point more than Black on those intersections. *The scores refer to the ko and dame intersections only*; only such intersections are denoted by scoring numbers in the diagrams. The other intersections can be ignored for endgame calculations because each of them always scores for the same player.

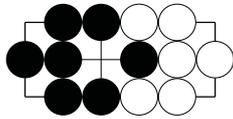
In this document, the word "ko" is an abbreviation for "basic endgame kos". The word "value" is an abbreviation for "deire value", which is the endgame value after Black moving first minus that one after White moving first.

## Variety of Aspects

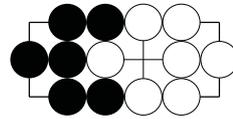
Before strategy is studied, first an overview is given on the variety of those aspects that play an important role for strategic planning. Later the examples are sorted by which aspects they have *initially* in their starting position.

## Ko Open for Black or White

Each ko is open for either Black or White.



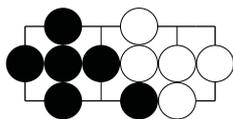
The ko is open for Black.



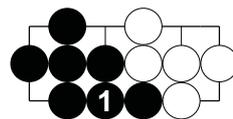
The ko is open for White.

## Black or White to Move

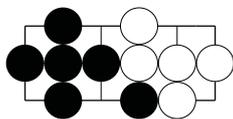
Either Black or White is the player having the turn and therefore the first to make a move in the current starting position.



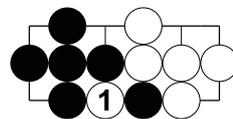
Black to move.



Black plays at 1.



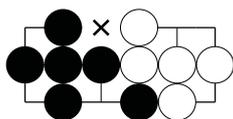
White to move.



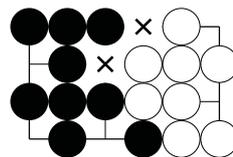
White plays at 1.

## Dame Parity

The number of dame can be either odd or even.



There is 1 dame. This is an odd number.



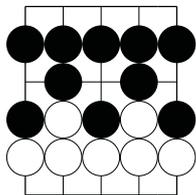
There are 2 dame. This is an even number.

## Difference of Numbers of Open Kos

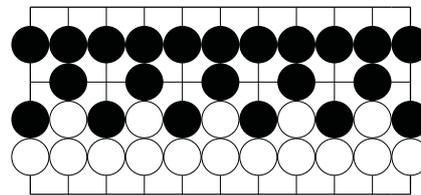
The difference of numbers of open kos is the number of kos open for Black minus the number of kos open for White. If there are more kos open for White, then the number of kos open for White minus the number of kos open for Black is calculated, i.e., for the sake of simplicity, the sign of the difference shall always be positive. Instead of the absolute size of the difference, mainly the remainder after its division by 3 is relevant strategically.

### Remainder 0 after Division by 3

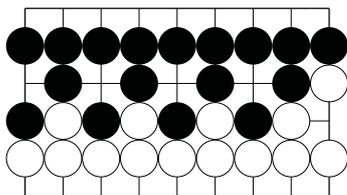
Dividing the difference of numbers of open kos by 3 gives the remainder 0. (The following integers have the remainder 0 after division by 3: 3, 6, 9, 12, 15,...)



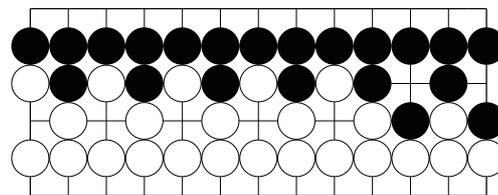
**Example A:** 3 kos are open for Black. 0 kos are open for White. The difference of numbers is  $3 - 0 = 3$ . The remainder after the division  $3 / 3$  is 0. (Why do we form  $3 / 3$ ? Because we divide the difference, which is 3, and because we always divide by 3. Why is the remainder 0? Because the result of the division  $3 / 3$  is the integer 1.)



**Example B:** 6 kos are open for Black. 0 kos are open for White. The difference of numbers is  $6 - 0 = 6$ . The remainder after the division  $6 / 3$  is 0. (Why do we form  $6 / 3$ ? Because we divide the difference, which is 6, and because we always divide by 3. Why is the remainder 0? Because the result of the division  $6 / 3$  is the integer 2.)

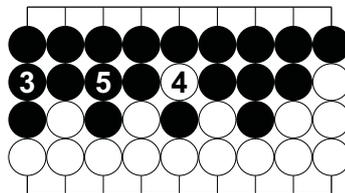
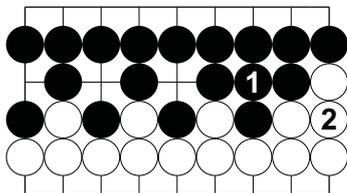


**Example C:** 4 kos are open for Black. 1 ko is open for White. The difference of numbers is  $4 - 1 = 3$ . The remainder after the division  $3 / 3$  is 0. (Note: What we divide is the difference - we do not divide the number of kos open for one particular player.)

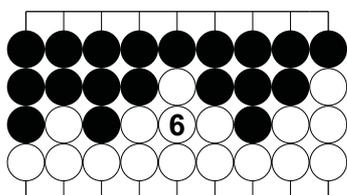


**Example D:** 5 kos are open for White. 2 kos are open for Black. The difference of numbers is  $5 - 2 = 3$ . The remainder after the division  $3 / 3$  is 0.

Now undoubtedly you wonder what do we gain from noticing that the remainder is 0 in each of these examples? The strategy is the same! Either player can fill a ko or else capture a ko until all the kos will be filled.



**Example C**, Black to move: After move 2, the position is reduced to example A.



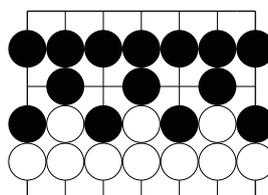
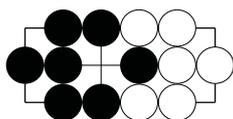
⑦ pass, ⑧ pass.

Firstly the players fill an equal number of excess kos. Secondly, the player with more open kos fills 2/3 of the remaining triples while the opponent fills 1/3.

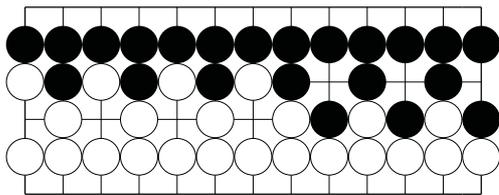
Then, in the triple of 3 kos open for Black, Black fills 2 while White captures and fills 1 ko.

### Remainder 1 after Division by 3

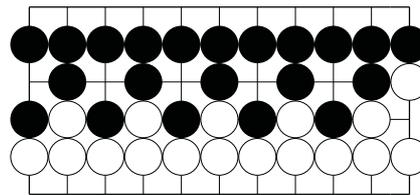
Dividing the difference of numbers of open kos by 3 gives the remainder 1. In other words, one player has 1 excess ko after ignoring as many pairs of one ko open for Black and one ko open for White and then ignoring as many triples of kos as possible. (The following integers have the remainder 1 after division by 3: 1, 4, 7, 10, 13,...)



**Example A:** 1 ko is open for Black. 0 kos are open for White. The difference of numbers is  $1 - 0 = 1$ . The remainder after the division  $1 / 3$  is 1. (Why do we form  $1 / 3$ ? Because we divide the difference, which is 1, and because we always divide by 3. Why is the remainder 1? Because the result of the division  $1 / 3$  is 0.333, which is the integer 0 plus the not yet divided remainder 1.)



**Example B:** 4 kos are open for Black. 0 kos are open for White. The difference of numbers is  $4 - 0 = 4$ . The remainder after the division  $4 / 3$  is 1. (Why do we form  $4 / 3$ ? Because we divide the difference, which is 4, and because we always divide by 3. Why is the remainder 1? Because the result of the division  $4 / 3$  is 1.333, which is the integer 1 plus the not yet divided remainder 1.)



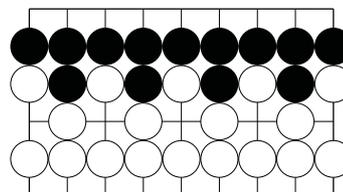
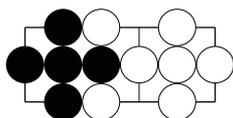
**Example C:** 4 kos are open for White. 3 kos are open for Black. The difference of numbers is  $4 - 3 = 1$ . The remainder after the division  $1 / 3$  is 1.

**Example D:** 5 kos are open for Black. 1 ko is open for White. The difference of numbers is  $5 - 1 = 4$ . The remainder after the division  $4 / 3$  is 1.

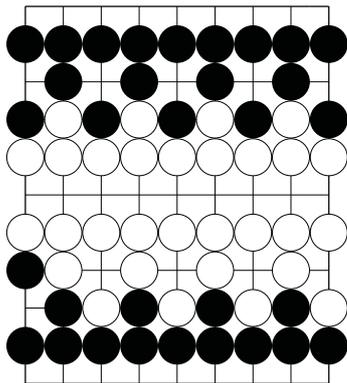
The class of positions with the remainder 1 is not so easy as the class of positions with remainder 0. Details of strategy are discussed later. In particular, it is not always correct to reduce the number of remaining kos to 1 immediately; filling a dame first might sometimes be necessary.

### Remainder 2 after Division by 3

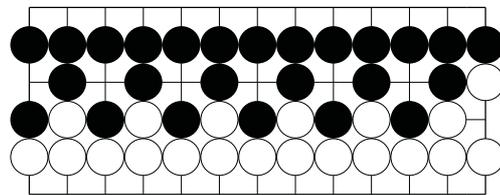
Dividing the difference of numbers of open kos by 3 gives the remainder 2. In other words, one player has 2 excess kos after ignoring as many pairs of one ko open for Black and one ko open for White and then ignoring as many triples of kos as possible. (The following integers have the remainder 2 after division by 3: 2, 5, 8, 11, 14,...)



**Example A:** 2 kos are open for White. 0 kos are open for Black. The difference of numbers is  $2 - 0 = 2$ . The remainder after the division  $2 / 3$  is 2. (Why do we form  $2 / 3$ ? Because we divide the difference, which is 2, and because we always divide by 3. Why is the remainder 2? Because the result of the division  $2 / 3$  is 0.666, which is the integer 0 plus the not yet divided remainder 2.)



**Example B:** 5 kos are open for White. 0 kos are open for Black. The difference of numbers is  $5 - 0 = 5$ . The remainder after the division  $5 / 3$  is 2. (Why do we form  $5 / 3$ ? Because we divide the difference, which is 5, and because we always divide by 3. Why is the remainder 2? Because the result of the division  $5 / 3$  is 1.666, which is the integer 1 plus the not yet divided remainder 2.)



**Example C:** 6 kos are open for Black. 4 kos are open for White. The difference of numbers is  $6 - 4 = 2$ . The remainder after the division  $2 / 3$  is 2.

**Example D:** 6 kos are open for Black. 1 ko is open for White. The difference of numbers is  $6 - 1 = 5$ . The remainder after the division  $5 / 3$  is 2.

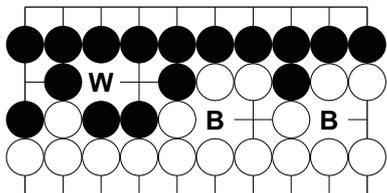
Also the class of positions with the remainder 2 is not so easy as the class of positions with remainder 0.

## Difference of Numbers of Ko Threats

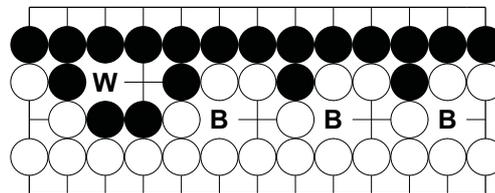
The difference of numbers of ko threats is the number of ko threats for the player having the greater number of open kos minus the number of ko threats for the opponent. If the difference is positive, then the player with the greater number of open kos has more ko threats than the opponent. If the difference is negative, then the player with smaller number of open kos leads on ko threats. If the difference is zero, then it matters who captures the ko first. However, as will be seen further below, in quite some ko and dame endgames, arithmetics are not that simple - the number of dame has to be taken into account as well.

(Here we assume that numbers of ko threats can be actually counted. Still one has to be careful: Playing a ko threat might also eliminate other ko threats; therefore one can only count the maximal number of ko threats that a player could play in the course of a ko fight. Instead of a simple threat - answer sequence, it is sometimes necessary to play a longer sequence like threat - answer - answer - answer; we count also such as only one ko threat. Then there are double ko threats: Either player playing some eliminates an opposing ko threat at the same time. Typically, double ko threats should be played before the ko fight starts.

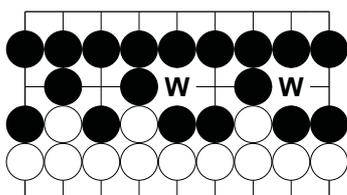
Furthermore, not everything that looks like a ko threat is one at every moment of a ko fight, e.g., if it could backfire. Such details are not the major topic of this document though and should be studied elsewhere.)



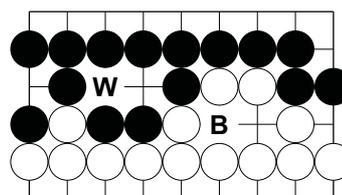
**Example A:** Black as the player with the greater number of open kos has 2 ko threats at the intersections B. White has 1 ko threat at the intersection W. The difference of numbers is  $2 - 1 = 1$ .



**Example B:** White as the player with the greater number of open kos has 1 ko threat at the intersection W. Black has 3 ko threats at the intersections B. The difference of numbers is  $1 - 3 = -2$ .



**Example C:** Black as the player with the greater number of open kos has 0 ko threats. White has 2 ko threats at the intersections W. The difference of numbers is  $0 - 2 = -2$ .

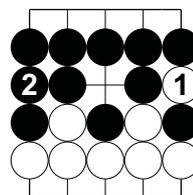
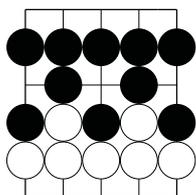


**Example D:** Black as the player with the greater number of open kos has 1 ko threat at the intersection B. White has 1 ko threat at the intersection W. The difference of numbers is  $1 - 1 = 0$ .

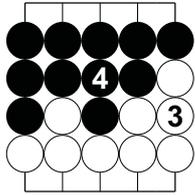
## Overview on the Possible Kinds of Ko Fights

### No Ko Fight

Although there are kos on the board, it is not necessary to fight them.



White to move, without dame, the difference of open kos gives the remainder 0, without ko threats.

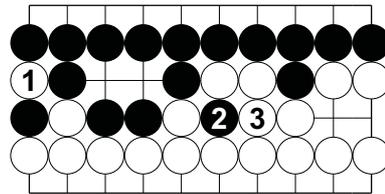
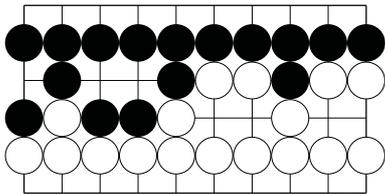


⑤ pass, ⑥ pass.

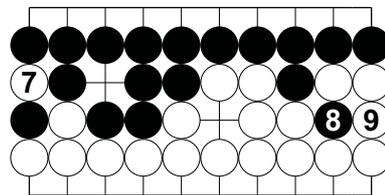
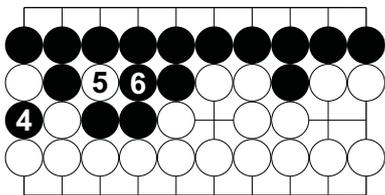
The players simply fill the kos as quickly as possible.

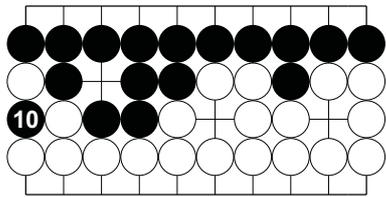
## Ordinary Ko Fight

The difference of ko threats decides the winner of the last ko.

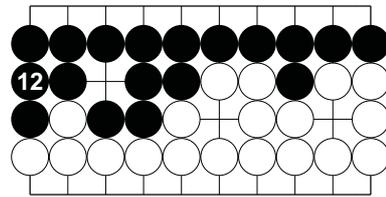


White to move, 0 dame, the difference of open kos gives the remainder 1, the difference of ko threats is 1.





11 pass.

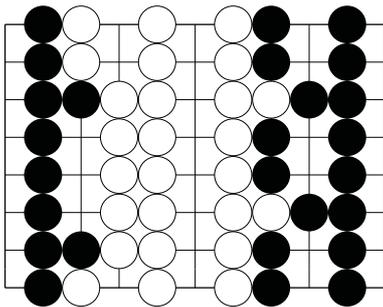


13 pass, 14 pass.

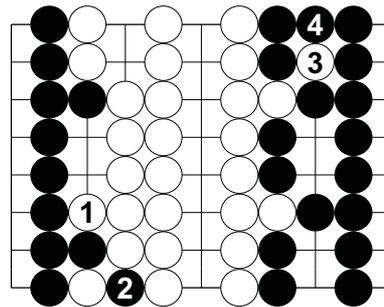
Black has had enough ko threats to win the ko by filling it.

## Dame Ko Fight

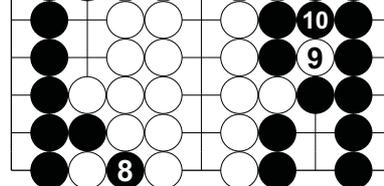
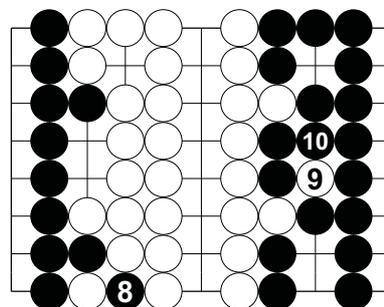
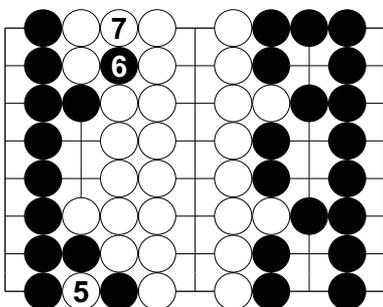
One player can also use dame instead of ko threats.

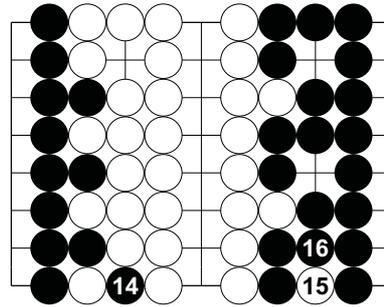
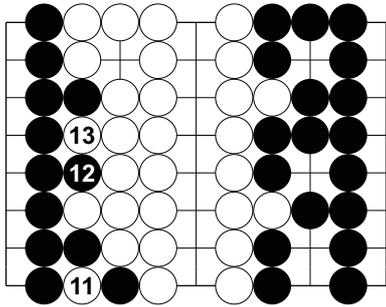


White to move, odd number of dame, the difference of open kos gives the remainder 1, the difference of ko threats is 2.

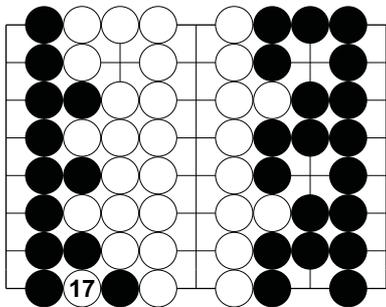


White does not only want to win the ko but also get the one extra odd dame.

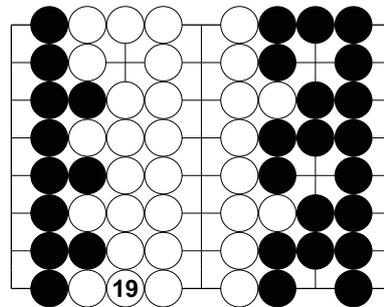




Black plays a dame instead of a ko threat.  
 White can answer by also filling a dame  
 because he still has a ko threat.



18 pass.



20 pass, 21 pass.

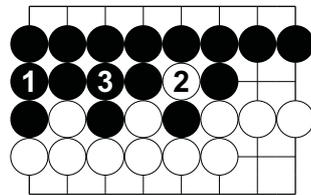
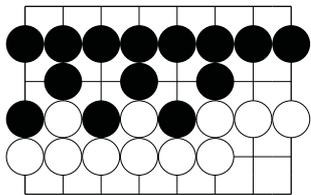
White has both filled the ko and got one  
 more dame than Black.

## Overview on the Possible Values of Moving First

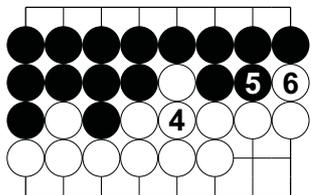
In ko and dame endgames, the value of Black moving first versus White moving first can be either 0, 2, or 4 points.

### Value 0

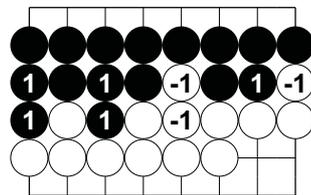
The score is the same regardless of whether Black or White moves first.



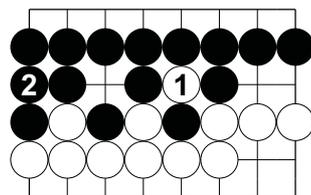
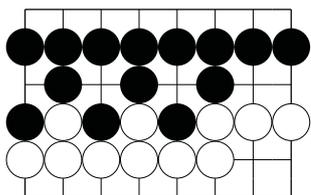
**Black to move.** Example with even number of dame, the difference of open kos gives the remainder 0, without ko threats.



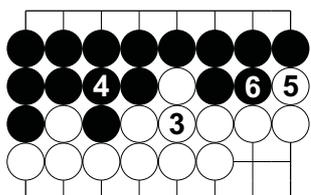
7 pass, 8 pass.



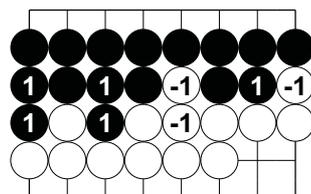
On the ko and dame intersections, the score is  $5 - 3 = 2$ .



**White to move.**



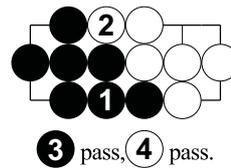
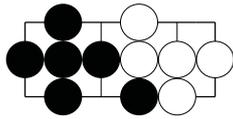
7 pass, 8 pass.



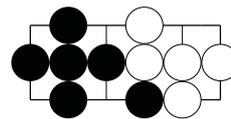
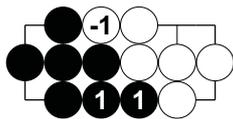
The score is  $5 - 3 = 2$ . This is the same as in the case Black to move.

## Value 2

Black moving first versus White moving first makes a 2 points difference to the score.

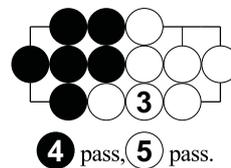
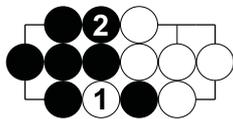


**Black to move.** Example with odd number of dame, the difference of open kos gives the remainder 1, without ko threats.



The score is  $2 - 1 = 1$ .

**White to move.**



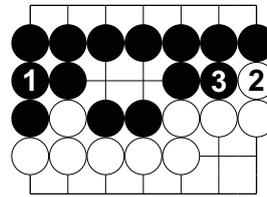
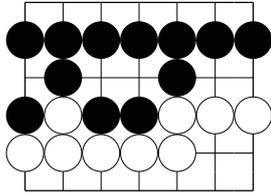
The score is  $1 - 2 = -1$ . This is a 2 points difference compared to the case Black to move.

Either player fills the ko to get 2 points on its intersections while the opponent fills a dame to get 1 point there. In either case, this amounts to a 1 point advantage for the first moving player. Thus the difference between Black moving first and White moving first is 2 points.

## Value 4

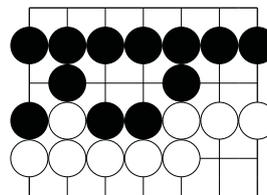
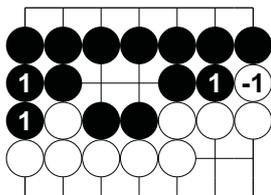
Black moving first versus White moving first makes a 4 points difference to the score. This case occurs only occasionally because a) usually there are dame in the endgame position, b) the dame parity needs to be *even*, and c) the right player needs more ko threats than roughly

the sum of the opponent's number of ko threats *plus half the number of dame* to win the dame ko fight.



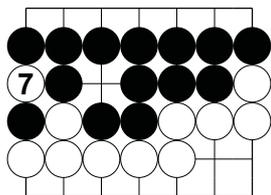
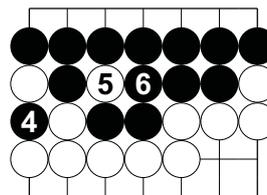
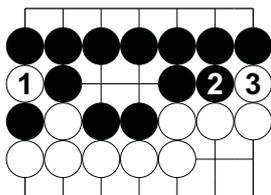
④ pass, ⑤ pass.

**Black to move.** Example with even number of dame, the difference of open kos gives the remainder 1, the difference of ko threats is -1.

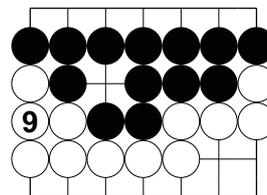


The score is  $3 - 1 = 2$ .

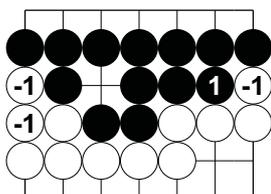
**White to move.**



⑧ pass.



⑩ pass, ⑪ pass.



What has happened? If Black fills the ko, he gets 2 points there. If White fills it, he gets those 2 points. 2 points for either player amount to a 4 points difference. White could raise the stake of the ko by keeping it open until all the dame had been filled.

The score is  $1 - 3 = -2$ . This is a 4 points difference compared to the case Black to move.

## Abbreviations and General Remarks

In the following chapters, the following abbreviations are used:

**king** = the player with initially the greater number of open kos

**slave** = the player with initially the smaller number of open kos

**D** = initial number of dame

**K** = difference of open kos = the initial numbers of the king's open kos minus the slave's open kos

**T** = difference of ko threats = the initial numbers of the king's ko threats minus the slave's ko threats

In general score formulae, the score is stated for Black as the king. If it is White, use the negative instead. E.g.,  $2/3 * K - 1$  for Black becomes  $-2/3 * K + 1$  for White. The contained  $2/3 * K$  seems big but recall that the king has played  $K$  more ko stones in them while the slave has played  $K$  more stones elsewhere on the board, which are not included in the formula. If they were included, then, e.g.,  $2/3 * K$  would be replaced by  $2/3 * K - K = -1/3 * K$ , i.e., it is actually the slave who gains points by turning one third of the kos initially open for the king into kos filled by the slave.

The stated standard strategies prefer simplicity to unnecessary complications, except that ko fights are actually being played, also the last ko is filled (even after the opponent's pass), and follow perfect play. If an opponent tries complications or trick plays, then a player needs to modify his strategy as well. This is not treated in this document but often the obvious answers are good enough: Answer a ko threat even if it has not been played after a ko capture. Fill a ko if the opponent has just captured another ko instead of filling some for himself. Finally, trivial things like pass as the last useful move are omitted.

Although it is not necessary to learn the score formulae, knowing them has the advantage that one's strategic planning is simplified: As soon as one has found a variation leading to the score, then one can stop further reading ahead because one already knows that the variation is a perfect play (unless it contains mistakes by both players). Afterwards it suffices to recognize and react to the opponent's possible non-standard strategies or trick plays.

# Types of Ko Fights without Dame

The endgame positions with kos but without dame can be classified into types depending on the remainder of the difference of numbers of open kos divided by 3, the first moving player, and the difference of numbers of ko threats.

When the endgame enters the ko and dame stage, it is scarce than there are no dame. Since strategy is simpler then, these types shall be listed first. For practical purposes, a profound knowledge of the types with dame is more important though.

## Type 1: Remainder 0

$K$  divided by 3 gives the remainder 0.

### General

Either player might be the one to move first.

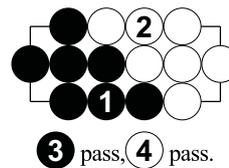
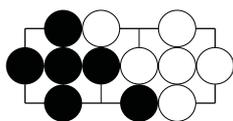
There is **no ko fight**.

The score is  $\frac{2}{3} * K$ .

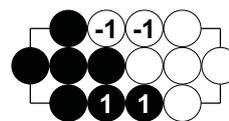
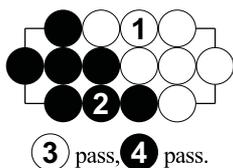
The value is **0**.

Strategy for each player: Fill a ko or else capture a ko until all the kos are filled.

### Example



We have  $K = 1 - 1 = 0$ . Applying the formula, we can already determine the score as  $\frac{2}{3} * 0 = 0$ . Black to move.



White to move.

Score after either Black or White to move:  $2 - 2 = 0$ .

## Type 2: Remainder 1, the King to Move

K divided by 3 gives the remainder 1. The king moves first.

### General

There is **no ko fight**.

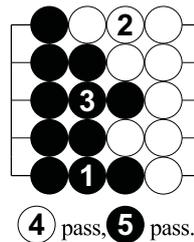
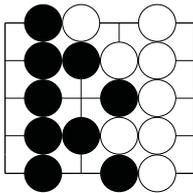
The score is  $\frac{2}{3} * K + \frac{4}{3}$ .

After the first move, which connects a ko, type 1 is reached.

Strategy for each player: Fill a ko or else capture a ko until all the kos are filled.

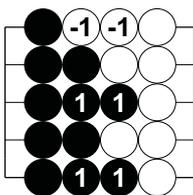
Strategy is independent of numbers of ko threats.

### Example



We have  $K = 2 - 1 = 1$ . Applying the formula, the score will be  $\frac{2}{3} * 1 + \frac{4}{3} = 2$ .

The king, Black, to move.



The score is  $4 - 2 = 2$ .

## Type 3: Remainder 1, the Slave to Move, $T > 0$

K divided by 3 gives the remainder 1. The slave moves first. The king has more ko threats.

### General

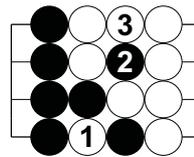
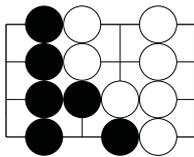
There is an **ordinary ko fight** won by the **king**.

The score is  $\frac{2}{3} * K + \frac{4}{3}$ , which is the same as in type 2.

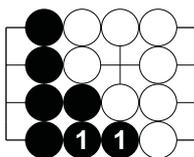
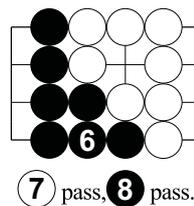
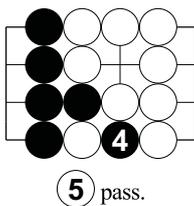
Strategy for each player: Choose the first possible option:

- 1) answer a ko threat
- 2) connect a ko
- 3) capture a ko
- 4) play a ko threat

### Example



We have  $K = 1$  and  $T = 1$ . White is the slave. Applying the formula, the score will be  $\frac{2}{3} * 1 + \frac{4}{3} = 2$ .



The score is  $2 - 0 = 2$ .

### Type 4: Remainder 1, the Slave to Move, $T \leq 0$

$K$  divided by 3 gives the remainder 1. The slave moves first. The slave has at least as many ko threats as the king.

#### General

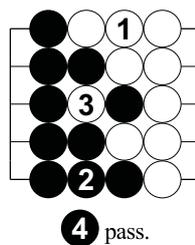
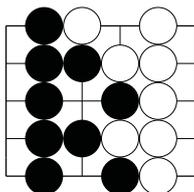
There is an **ordinary ko fight** won by the **slave**.

The score is  $\frac{2}{3} * K - \frac{8}{3}$ .

Strategy for each player: Choose the first possible option:

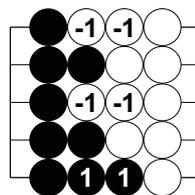
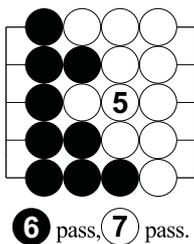
- 1) answer a ko threat
- 2) connect a ko
- 3) capture a ko
- 4) play a ko threat

## Example



We have  $K = 2 - 1 = 1$  and  $T = 0$ . White is the slave. Applying the formula, the score will be  $\frac{2}{3} * 1 - \frac{8}{3} = -2$ .

This is a very short ko fight because, already after White's first capture of the ko with the play 3, Black does not have any ko threat.



The score is  $2 - 4 = -2$ .

## Values of Types 2 to 4

### General

The combination of types 2 + 3 means remainder 1 and  $T > 0$  and has the value **0**.

The combination of types 2 + 4 means remainder 1 and  $T \leq 0$  and has the value **4**.

### Explanation

Combining two types means *either* player to move. The score after the king moves first is compared with the score after the slave moves first by forming the difference of the two scores.

E.g., consider the example shown for type 3, where White to move leads to the score 2. Now use the same example position for type 2, i.e., with Black as the king to move. You will find

that also this leads to the score 2. Hence the value of the initial example position is the difference of these scores:  $2 - 2 = 0$ . This value occurs in the combination of types  $2 + 3$ .

E.g., in the example shown for the types 2 and 4, Black to move leads to the score 2 while White to move leads to the score -2. The value of the initial example position is, by definition, the difference of these scores:  $2 - (-2) = 4$ . This value occurs in the combination of types  $2 + 4$ .

The ko threat difference determines whether the initial position has the value 0 or 4. As a practical consequence, before the endgame reaches the basic endgame ko stage, in principle it is better to have more ko threats than the opponent. One should recall this already during the middle game and the earlier endgame. E.g., if one can defend so that the opponent does not have nearby ko threats, then this is preferable to leaving behind aji.

## Type 5: Remainder 2, the Slave to Move

$K$  divided by 3 gives the remainder 2. The slave moves first.

### General

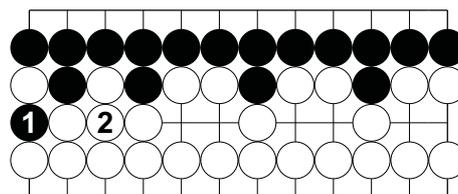
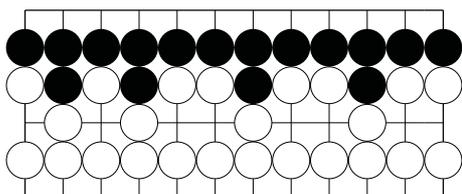
There is **no ko fight**.

The score is  $\frac{2}{3} * K - \frac{4}{3}$ .

Strategy for each player: Fill a ko or else capture a ko until all the kos are filled.

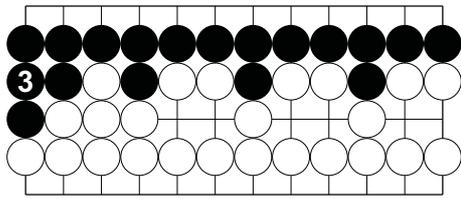
Strategy is independent of numbers of ko threats.

### Example

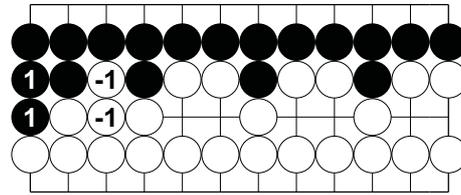


We have  $K = 2 - 0 = 2$ . Applying the formula negated for White as the king, the score will be  $-(\frac{2}{3} * 2 - \frac{4}{3}) = -0 = 0$ .

Black as the slave moves first.



④ pass, ⑤ pass.



Black has not needed any of his ko threats. The score is  $2 - 2 = 0$ .

## Type 6: Remainder 2, the King to Move, $T > 0$

$K$  divided by 3 gives the remainder 2. The king moves first and has more ko threats.

### General

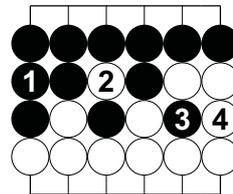
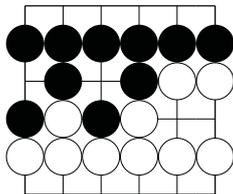
There is an **ordinary ko fight** won by the **king**.

The score is  $\frac{2}{3} * K + \frac{8}{3}$ .

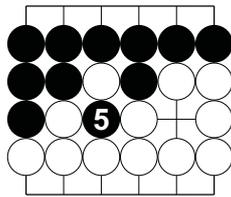
Strategy for each player: Choose the first possible option:

- 1) answer a ko threat
- 2) connect a ko
- 3) capture a ko
- 4) play a ko threat

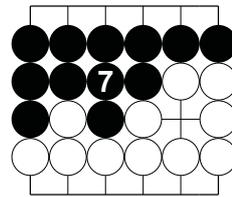
### Example



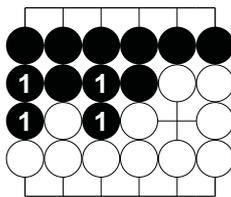
We have  $K = 2$  and  $T = 1$ . Black is the king.  
 Applying the formula, the score will be  $\frac{2}{3} * 2 + \frac{8}{3} = 4$ .



⑥ pass.



⑧ pass, ⑨ pass.



The score is  $4 - 0 = 4$ .

## Type 7: Remainder 2, the King to Move, $T \leq 0$

$K$  divided by 3 gives the remainder 2. The king moves first. The slave has at least as many ko threats as the king.

### General

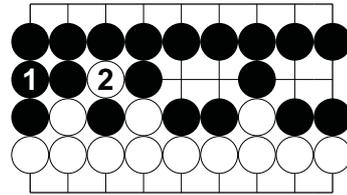
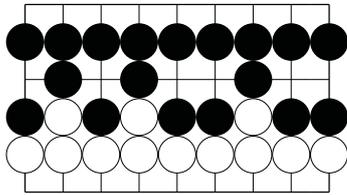
There is an **ordinary ko fight** won by the **slave**.

The score is  $\frac{2}{3} * K - \frac{4}{3}$ , which is the same as in type 5.

Strategy for each player: Choose the first possible option:

- 1) answer a ko threat
- 2) connect a ko
- 3) capture a ko
- 4) play a ko threat

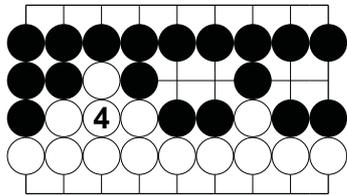
### Example



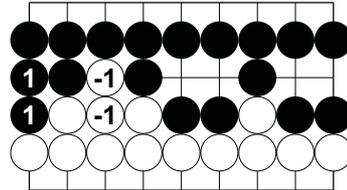
3 pass.

We have  $K = 2$  and  $T = -2$ . Black is the king.  
Applying the formula, the score will be  $2/3$   
 $* 2 - 4/3 = 0$ .

The ko threats are White's - Black has none.



5 pass, 6 pass.



The score is  $2 - 2 = 0$ .

## Values of Types 5 to 7

### General

The combination of types 5 + 6 means remainder 2 and  $T > 0$  and has the value **4**.

The combination of types 5 + 7 means remainder 2 and  $T \leq 0$  and has the value **0**.

## Summary of the Types without Dame

### Conditions, Ko Fights, and Score

Type	Remainder	To Move	T	Ko Fight	Ko Fight Winner	Black's Score - $2/3 * K$
1	0	either	any	none		0
2	1	king	any	none		4/3
3		slave	>0	ordinary	king	4/3
4			≤0		slave	-8/3
5		slave	any	none		-4/3
6	2	king	>0	ordinary	king	8/3
7			≤0		slave	-4/3

### Values

Types	Remainder	T	Value
1	0	any	0
2+3	1	>0	0
2+4		≤0	4
5+6	2	>0	4
5+7		≤0	0

The value 4 occurs when, in the involved ordinary ko fight type, the ko threat difference favours the player to move first.

### Strategy

If there is **no ko fight**, then either player's strategy is: Fill a ko or else capture a ko until all the kos are filled.

If there is an **ordinary ko fight**, then either player's strategy is to choose the first possible option:

- 1) answer a ko threat
- 2) connect a ko

- 3) capture a ko
- 4) play a ko threat

## **Symmetry**

The types 2 to 4 are almost in perfect symmetry with the types 5 to 7: The roles of king or slave to move and the values are swapped and the score modifiers are negated. The only exception is that, in a type with an ordinary ko fight, the ko threat difference 0 favours the slave.

# Types of Ko Fights with Dame

The endgame positions with kos and dame can be classified into types depending on the remainder of the difference of numbers of open kos divided by 3, the dame parity, the first moving player, and the difference of numbers of ko threats.

## Type 8: Remainder 0, Dame Parity Even

$K$  divided by 3 gives the remainder 0. The dame parity is even.

### General

Either player might be the one to move first.

There is **no ko fight**.

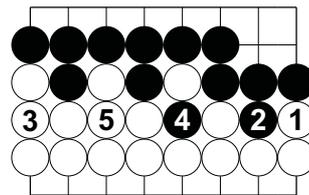
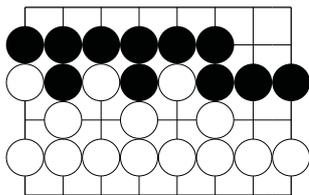
The score is  $\frac{2}{3} * K$ .

The value is **0**.

Strategy for each player: Choose the first possible option:

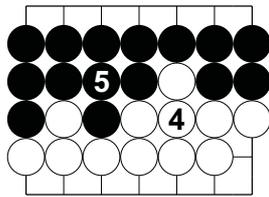
- 1) fill a dame
- 2) connect a ko
- 3) capture a ko

### Example

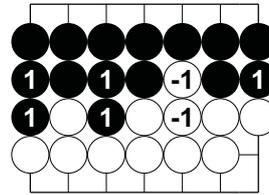


White to move. We have  $K = 3$ ,  $D = 2$ , and the dame parity even. Applying the formula negated for White as the king, the score will be  $-(\frac{2}{3} * 3) = -2$ .





⑥ pass, ⑦ pass.



The score is  $5 - 2 = 3$ .

## Type 10: Remainder 0, Dame Parity Odd, White to Move

$K$  divided by 3 gives the remainder 0. The dame parity is odd. White moves first.

### General

There is **no ko fight**.

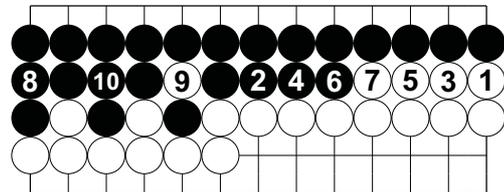
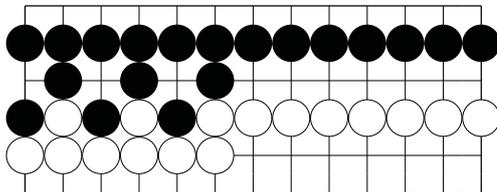
The score is  $\frac{2}{3} * K - 1$ .

If White is the king, then the score is  $-\frac{2}{3} * K - 1$ .

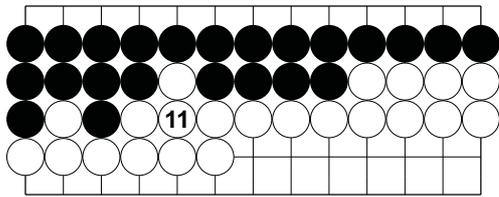
Strategy for each player: Choose the first possible option:

- 1) fill a dame
- 2) connect a ko
- 3) capture a ko

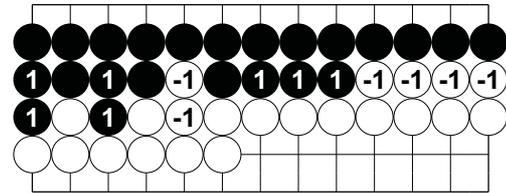
### Example



We have  $K = 3$ ,  $D = 7$ , and the dame parity odd. Applying the formula, the score will be  $\frac{2}{3} * 3 - 1 = 1$ .



12 pass, 13 pass.



The score is  $7 - 6 = 1$ .

## Values of Types 9 and 10

The combination of types  $9 + 10$  means remainder 0 and dame parity odd and has the value 2.

Whoever gets the first dame makes 1 extra point there. 1 point for Black versus 1 point for White explains the value 2.

## Summary of the Types with Dame and Remainder 0

### Conditions, Ko Fights, and Score

Type	Dame Parity	To Move	T	Ko Fight	Black's Score - $\frac{2}{3} * K$
8	even	either	any	none	0
9	odd	Black			1
10		White			-1

### Values

Types	Dame Parity	Value
8	even	0
9+10	odd	2

### Strategy

Strategy for each player: Choose the first possible option:

- 1) fill a dame
- 2) connect a ko
- 3) capture a ko

### Symmetry

The types 9 and 10 are symmetrical with respect to the score modifier.

# Type 11: Remainder 1, Dame Parity Even, the King to Move

K divided by 3 gives the remainder 1. The dame parity is even. The king moves first.

## General

There is **no ko fight**.

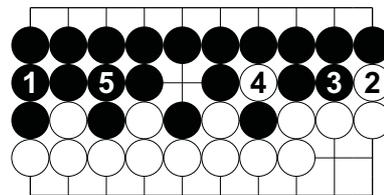
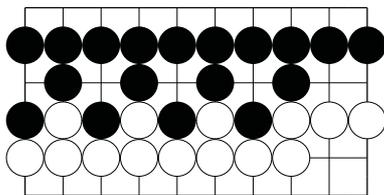
The score is  $\frac{2}{3} * K + \frac{4}{3}$ .

Strategy for each player: Choose the first possible option:

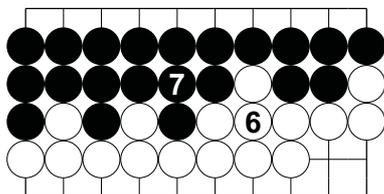
- 1) If it is the king's first move, connect a ko.
- 2) fill a dame
- 3) connect a ko
- 4) capture a ko

After the first move, the position is reduced to type 8, provided there are still some kos left.

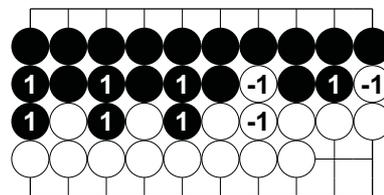
## Example



We have  $K = 4$ ,  $D = 2$ , and the dame parity even. Black is the king. Applying the formula, the score will be  $\frac{2}{3} * 4 + \frac{4}{3} = 4$ .



8 pass, 9 pass.



The score is  $7 - 3 = 4$ .

## Type 12: Remainder 1, Dame Parity Even, the Slave to Move, $T > D/2$

K divided by 3 gives the remainder 1. The dame parity is even. The slave moves first. The king has a number of ko threats that exceeds the slave's number of ko threats by more than half the number of dame.

On the 19x19 board, this type occurs only occasionally because typically there will be a rather big number of dame and so having a great enough excess of ko threats is hard.

### General

There is a **dame ko fight** won by the **king**.

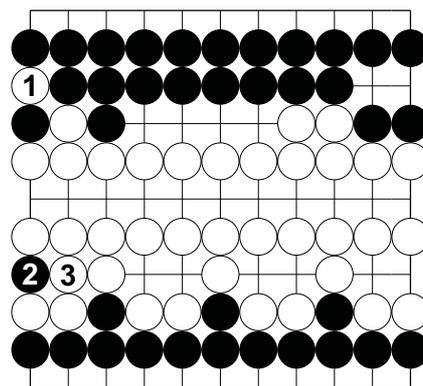
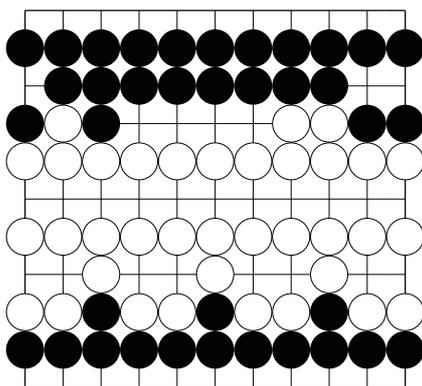
The score is  $\frac{2}{3} * K + \frac{4}{3}$ .

Strategy for each player: Choose the first possible option:

- 1) answer a ko threat
- 2) as the king, answer filling a dame by filling a dame
- 3) connect a ko
- 4) capture a ko
- 5) play a ko threat
- 6) as the slave, fill a dame after a ko capture

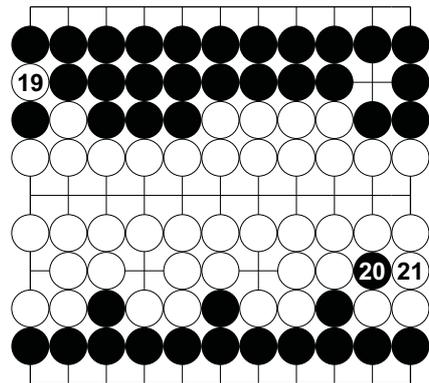
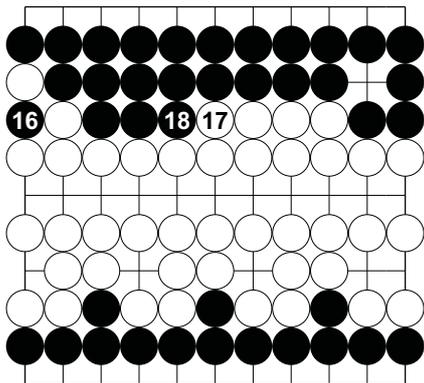
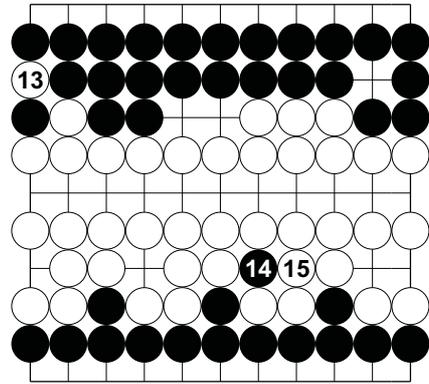
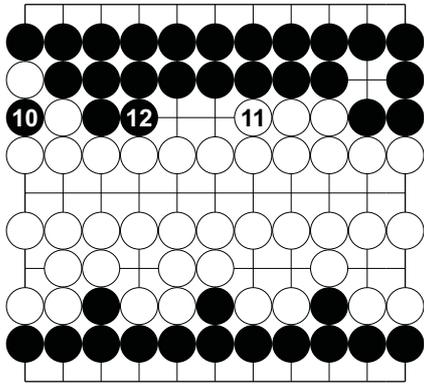
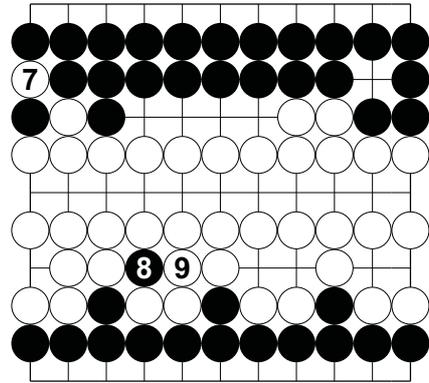
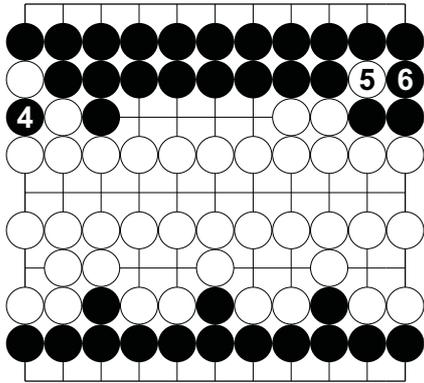
The slave can use dame instead of ko threats. The king cannot do so because then the slave would improve on the score by connecting the last ko. During the ko fight, all dame are filled.

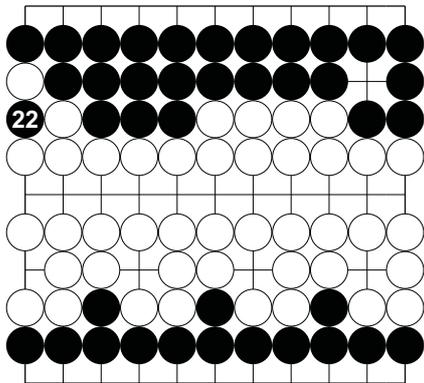
### Example



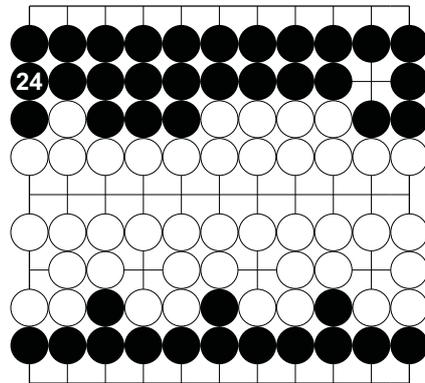
We have  $K = 1$ , the king Black,  $D = 4$ , the dame parity even,  $T = 4 - 1 = 3$ . The type's condition  $T > D/2$  is fulfilled:  $3 > 4/2$ .

Applying the formula, the score will be  $\frac{2}{3} * 1 + \frac{4}{3} = 2$ .

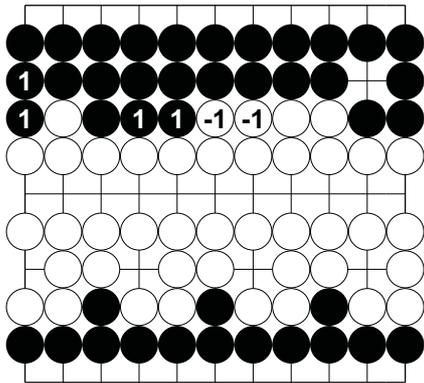




23 pass.



25 pass, 26 pass.



The score is  $4 - 2 = 2$ .

### Type 13A: Remainder 1, Dame Parity Even, the Slave to Move, $0 < T \leq D/2$

$K$  divided by 3 gives the remainder 1. The dame parity is even. The slave moves first. The king has more ko threats but his excess of ko threats is not greater than half the number of dame.

This is an ordinarily occurring ko threat situation.

#### General

There is an **ordinary ko fight** won by the **king**, but the king may choose to avoid it. **Either** player might connect the last ko.

The score is  $\frac{2}{3} * K - \frac{2}{3}$ .

Strategy for each player if the king chooses the ordinary ko fight: Choose the first possible option:

- 1) answer a ko threat
- 2) connect ko
- 3) capture ko
- 4) play a ko threat
- 5) fill a dame

The slave cannot force a dame ko fight because the king connects the last ko if the slave fills a dame. The king cannot force a dame ko fight about the last ko because the slave connects the last ko if the king fills a dame.

Strategy for each player if the king chooses to avoid the ordinary ko fight:

- 1) Strategy until the last ko: Choose the first possible option:
  - 1.1) connect a ko
  - 1.2) capture a ko
- 2) Strategy when only one ko remains: Choose the first possible option:
  - 2.1) as the slave's first move in (2), capture the ko
  - 2.2) connect the ko
  - 2.3) fill a dame

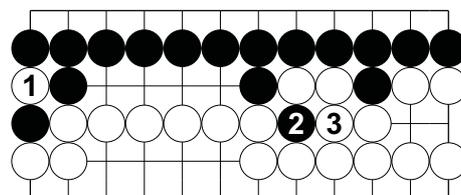
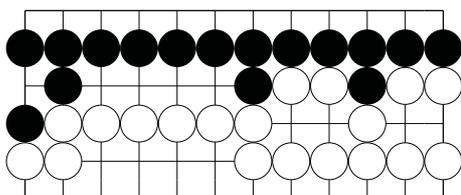
If one prefers a simplified view, one can use the following:

Simplified strategy for each player: Choose the first possible option:

- 1) connect a ko
- 2) capture a ko
- 3) fill a dame

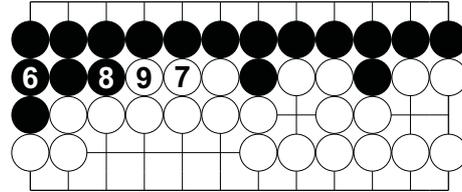
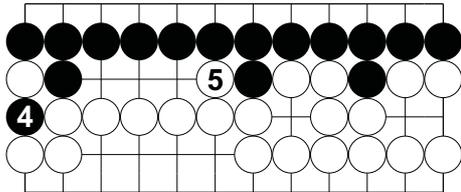
For the last ko, there are these two principle possibilities: either [the slave fills a dame - the king connects the ko - the slave fills a dame] or [the slave captures the ko - the king fills a dame - the slave connects the ko - the king fills a dame]. In both variations, the local score made on the intersections of the last ko and two involved dame is 0 and afterwards the parity of the number remaining dame is even. Therefore it does not matter which player wins the last ko with the opponent getting to play dame in the meantime, i.e., unless type 12 is created. The latest moment for the slave to start capturing the ko is when otherwise T would become greater than half the number of remaining dame. Such a type conversion would lose the slave 2 points.

## Example

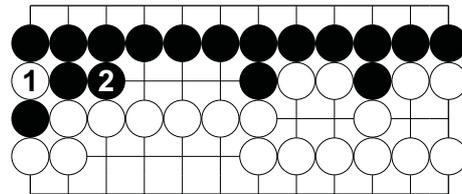
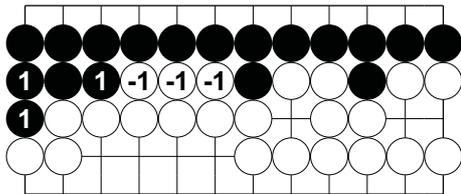


We have  $K = 1$ , the king Black,  $D = 4$ , the dame parity even,  $T = 2$ . The type's condition  $0 < T \leq D/2$  is fulfilled:  $0 < 2 \leq 4/2$ . Applying the formula, the score will be  $2/3 * 1 - 2/3 = 0$ . White as the slave moves first.

**Variation 1:** Black chooses the ordinary ko fight.

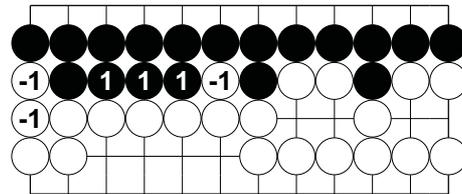
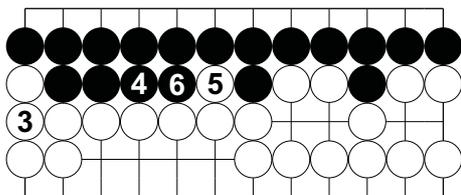


10 pass, 11 pass.



The score is  $3 - 3 = 0$ .

**Variation 2:** Black avoids the ordinary ko fight.



7 pass, 8 pass.

The score is  $3 - 3 = 0$ .

### Type 13B: Remainder 1, Dame Parity Even, the Slave to Move, $-D/2 < T \leq 0$

$K$  divided by 3 gives the remainder 1. The dame parity is even. The slave moves first. The slave has more or an equal number of ko threats but his excess of ko threats is not greater than or equal to half the number of dame.

This is an ordinarily occurring ko threat situation.

## General

There is an **ordinary ko fight** won by the **slave**, but the slave may choose to avoid it. **Either** player might connect the last ko.

The score is  $2/3 * K - 2/3$ .

Strategy for each player if the slave chooses the ordinary ko fight: Choose the first possible option:

- 1) answer a ko threat
- 2) connect ko
- 3) capture ko
- 4) play a ko threat
- 5) fill a dame

The king cannot force a dame ko fight about the last ko because the slave connects the last ko if the king fills a dame. The slave cannot force a dame ko fight because the king connects the last ko if the slave fills a dame.

Strategy for each player if the slave chooses to avoid the ordinary ko fight:

- 1) Strategy until the last ko: Choose the first possible option:
  - 1.1) connect a ko
  - 1.2) capture a ko
- 2) Strategy when only one ko remains: Choose the first possible option:
  - 2.1) as the king's first move in (2), connect the ko
  - 2.2) fill a dame

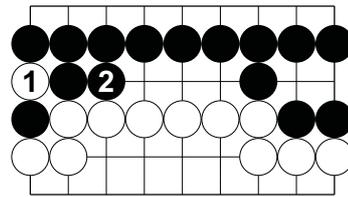
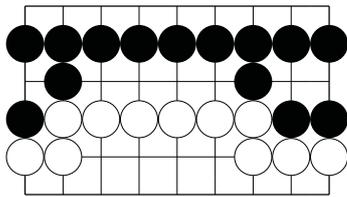
If one prefers a simplified view, one can use the following:

Simplified strategy for each player: Choose the first possible option:

- 1) connect a ko
- 2) capture a ko
- 3) fill a dame

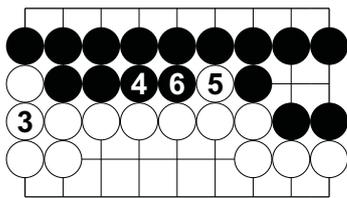
For the last ko, there are these two principle possibilities: either [the slave fills a dame - the king connects the ko - the slave fills a dame] or [the slave captures the ko - the king fills a dame - the slave connects the ko - the king fills a dame]. In both variations, the local score made on the intersections of the last ko and two involved dame is 0 and afterwards the parity of the number remaining dame is even. Therefore it does not matter which player wins the last ko with the opponent getting to play dame in the meantime, i.e., unless type 14 is created. The latest moment for the king to connect the ko is when otherwise T would become smaller than or equal to half the number of remaining dame. Such a type conversion would lose the king 2 points.

## Example

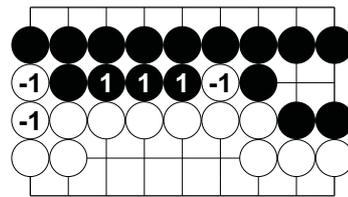


We have  $K = 1$ , the king Black,  $D = 4$ , the dame parity even,  $T = -1$ . The type's condition  $-D/2 < T \leq 0$  is fulfilled:  $-4/2 < -1 \leq 0$ . Applying the formula, the score will be  $2/3 * 1 - 2/3 = 0$ . White as the slave moves first.

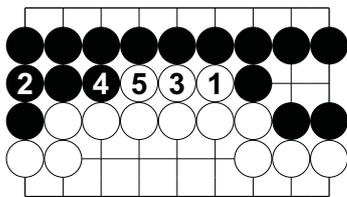
**Variation 1:** White chooses the ordinary ko fight. The ko fight is short because Black does not even have one ko threat; he admits defeat already at move 2.



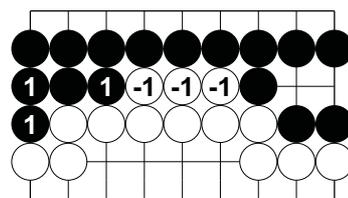
7 pass, 8 pass.



The score is  $3 - 3 = 0$ .



6 pass, 7 pass.



The score is  $3 - 3 = 0$ .

**Variation 2:** White avoids the ordinary ko fight.

### Type 14: Remainder 1, Dame Parity Even, the Slave to Move, $T \leq -D/2$

$K$  divided by 3 gives the remainder 1. The dame parity is even. The slave moves first. The slave has a number of ko threats that exceeds the king's number of ko threats by at least half the number of dame.

On the 19x19 board, this type occurs only occasionally.

## General

There is a **dame ko fight** won by the **slave**.

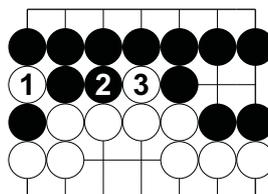
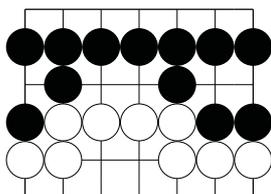
The score is  $\frac{2}{3} * K - \frac{8}{3}$ .

Strategy for each player: Choose the first possible option:

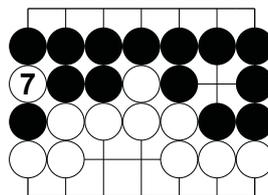
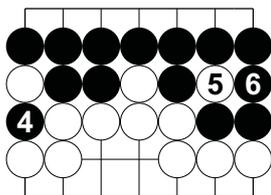
- 1) answer a ko threat
- 2) as the slave, answer filling a dame by filling a dame
- 3) connect a ko
- 4) capture a ko
- 5) play a ko threat
- 6) as the king, fill a dame after a ko capture

The king can use dame instead of ko threats. The slave cannot do so because then the king would improve on the score by connecting the last ko. During the ko fight, all dame are filled.

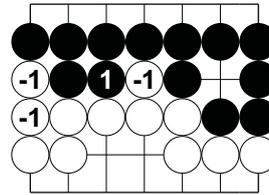
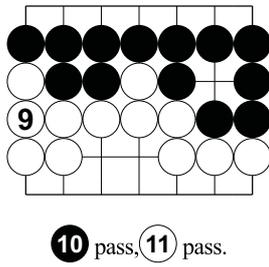
## Example



We have  $K = 1$ , the king Black,  $D = 2$ , the dame parity even,  $T = -1$ . The type's condition  $T \leq -D/2$  is fulfilled:  $-1 \leq -2/2$ . Applying the formula, the score will be  $\frac{2}{3} * 1 - \frac{8}{3} = -2$ .



8 pass.



The score is  $1 - 3 = -2$ .

## Values of Types 11 to 14

The combination of types 11 + 12 means remainder 1, dame parity even, and  $T > D/2$  and has the value **0**.

The combination of types 11 + 13A means remainder 1, dame parity even, and  $0 < T \leq D/2$  and has the value **2**.

The combination of types 11 + 13B means remainder 1, dame parity even, and  $-D/2 < T \leq 0$  and has the value **2**.

The combination of types 11 + 14 means remainder 1, dame parity even, and  $T \leq -D/2$  and has the value **4**.

## Type 15: Remainder 1, Dame Parity Odd, the King to Move, $T > (D-1)/2$

$K$  divided by 3 gives the remainder 1. The dame parity is odd. The king moves first. The king has a number of ko threats that exceeds the slave's number of ko threats by more than rounded down half the number of dame.

On the 19x19 board, this type occurs only occasionally.

### General

If  $D > 1$ , there is a **dame ko fight** won by the **king**. If  $D = 1$ , it is an ordinary ko fight.

The score is  $\frac{2}{3} * K + \frac{7}{3}$ .

Strategy for each player: Choose the first possible option:

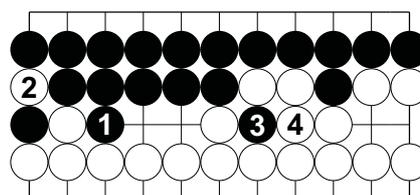
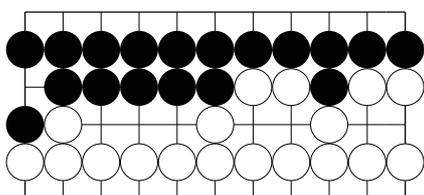
- 1) as the king's first move, fill a dame
- 2) answer a ko threat
- 3) as the king, answer filling a dame by filling a dame
- 4) connect a ko

- 5) capture a ko
- 6) play a ko threat
- 7) as the slave, fill a dame after a ko capture

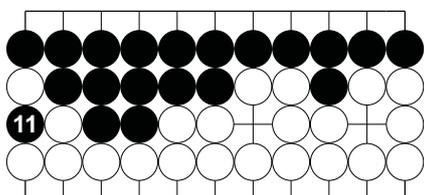
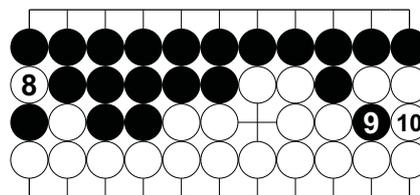
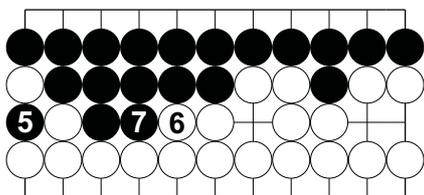
The slave can use dame instead of ko threats. The king cannot do so because then the slave would improve on the score by connecting the last ko. During the ko fight, all remaining dame are filled.

After the king's first move, the position is reduced to type 12 if  $D > 1$ , else to type 3.

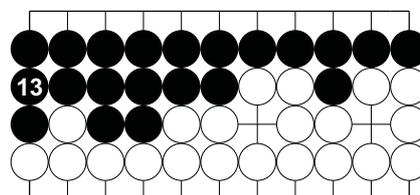
### Example



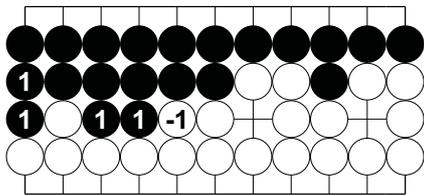
We have  $K = 1$ , the king Black,  $D = 3$ , the dame parity odd,  $T = 2$ . The type's condition  $T > (D-1)/2$  is fulfilled:  $2 > (3-1)/2$ .  
 Applying the formula, the score will be  $2/3 * 1 + 7/3 = 3$ .



12 pass.



14 pass, 15 pass.



The score is  $4 - 1 = 3$ .

## Type 16: Remainder 1, Dame Parity Odd, the King to Move, $T \leq (D-1)/2$

$K$  divided by 3 gives the remainder 1. The dame parity is odd. The king moves first. The king does not have a number of ko threats that exceeds the slave's number of ko threats by more than rounded down half the number of dame. It might also be the slave who has more ko threats.

This is the ordinarily occurring ko threat situation. Even if the slave had an excess of ko threats greater than half the number of dame, it would be useless because the king simply avoids a ko fight by starting with filling the excess ko, what leads to the remainder 0.

### General

There is **no ko fight**.

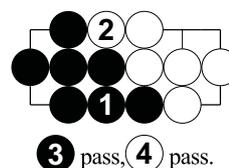
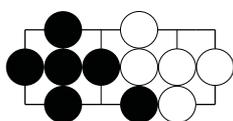
The score is  $\frac{2}{3} * K + \frac{1}{3}$ .

Strategy for each player: Choose the first possible option:

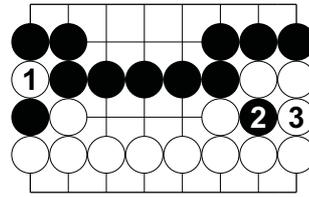
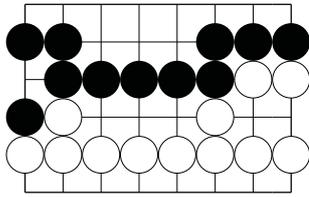
- 1) as the king's first move, connect a ko.
- 2) fill a dame
- 3) connect a ko
- 4) capture a ko

After the king's first move, the position is reduced to type 9 or 10, if then still a ko remains.

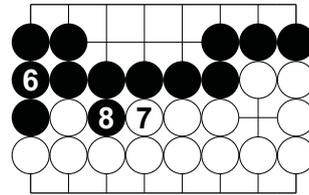
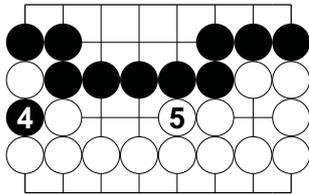
### Example



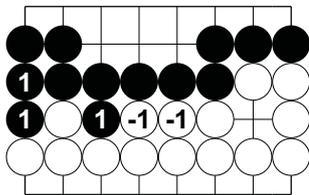




We have  $K = 1$ , the king Black,  $D = 3$ , the dame parity odd,  $T = 1$ . The type's condition  $T > 0$  is fulfilled. Applying the formula, the score will be  $2/3 * 1 + 1/3 = 1$ .



9 pass, 10 pass.



The score is  $3 - 2 = 1$ .

## Type 18: Remainder 1, Dame Parity Odd, the Slave to Move, $T \leq 0$

$K$  divided by 3 gives the remainder 1. The dame parity is odd. The slave moves first. The slave has at least as many ko threats as the king.

### General

This is an **ordinary ko fight** won by the **slave**.

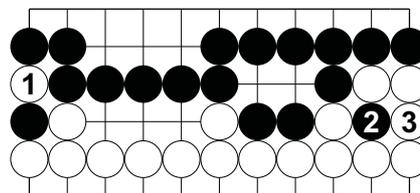
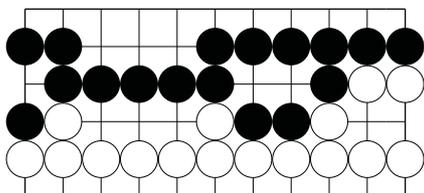
The score is  $2/3 * K - 5/3$ .

Strategy for each player: Choose the first possible option:

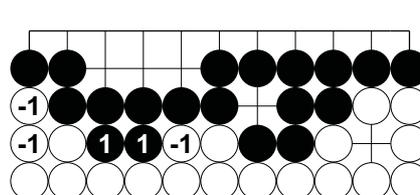
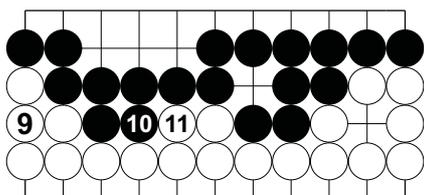
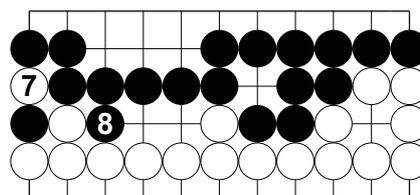
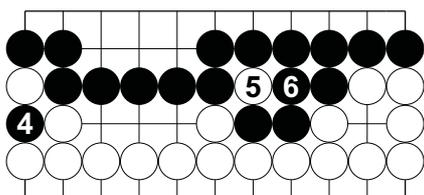
- 1) answer a ko threat
- 2) connect ko
- 3) capture ko
- 4) play a ko threat
- 5) fill a dame

The king cannot force a dame ko fight because the slave connects the last ko as soon as the king fills a dame. - The slave cannot force a dame ko fight about the last ko because the king can simply continue to fill dame instead of further attempting to win the ko after he has run out of ko threats. The king loses the ko fight but this does not mean that he would also have to concede the extra odd dame to the slave. In fact, the king could fill it in his first move, admitting defeat in the ko fight before taking part in it at all.

### Example



We have  $K = 1$ , the king Black,  $D = 3$ , the dame parity odd,  $T = 0$ . The type's condition  $T \leq 0$  is fulfilled. Applying the formula, the score will be  $2/3 * 1 - 5/3 = -1$ .



12 pass, 13 pass.

The score is  $2 - 3 = -1$ .

## Values of Types 15 to 18

The combination of types 15 + 17 means remainder 1, dame parity odd, and  $T > (D-1)/2$  and has the value **2**.

The combination of types 16 + 17 means remainder 1, dame parity odd, and  $0 < T \leq (D-1)/2$  and has the value **0**.

The combination of types 16 + 18 means remainder 1, dame parity odd, and  $T \leq 0$  and has the value **2**.

# Summary of the Types with Dame and Remainder 1

## Conditions, Ko Fights, and Score

Type	Dame Parity	To Move	T	Ko Fight	Ko Fight Winner	Black's Score - $\frac{2}{3} * K$
11	even	king	any	none		4/3
12		slave	$T > D/2$	<b>dame</b>	king	
13A			$0 < T \leq D/2$	<b>ordinary</b>	slave	-2/3
13B			$-D/2 < T \leq 0$			
14		$T \leq -D/2$	<b>dame</b>		-8/3	
15	odd	king	$T > (D-1)/2$	<b>dame</b>	king	7/3
16			$T \leq (D-1)/2$	none		1/3
17		slave	$T > 0$	<b>ordinary</b>	king	
18			$T \leq 0$		slave	-5/3

## Values

Types	Dame Parity	T	Value
11+12	even	$T > D/2$	0
11+13A		$0 < T \leq D/2$	2
11+13B		$-D/2 < T \leq 0$	
11+14		$T \leq -D/2$	4
15+17	odd	$T > (D-1)/2$	2
16+17		$0 < T \leq (D-1)/2$	0
16+18		$T \leq 0$	2

## Type 19: Remainder 2, Dame Parity Even, the Slave to Move

K divided by 3 gives the remainder 2. The dame parity is even. The slave moves first.

### General

There is **no ko fight**.

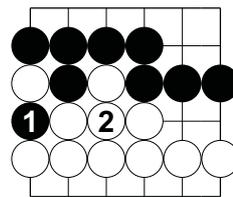
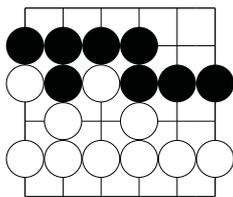
The score is  $\frac{2}{3} * K - \frac{4}{3}$ .

Strategy for each player: Choose the first possible option:

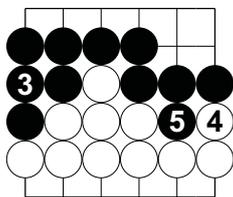
- 1) connect a ko
- 2) capture a ko
- 3) fill a dame

After the slave's first move, the position is reduced to type 11.

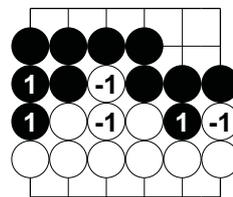
### Example



We have  $K = 2$ , the king White,  $D = 2$ , the dame parity even. Applying the formula negated for White as the king, the score will be  $-(\frac{2}{3} * 2 - \frac{4}{3}) = -0 = 0$ .



⑥ pass, ⑦ pass.



The score is  $3 - 3 = 0$ .

## Type 20: Remainder 2, Dame Parity Even, the King to Move, $T > D/2$

K divided by 3 gives the remainder 2. The dame parity is even. The king moves first. The king has a number of ko threats that exceeds the slave's number of ko threats by more than half the number of dame.

On the 19x19 board, this type occurs only occasionally.

### General

There is a **dame ko fight** won by the **king**.

The score is  $\frac{2}{3} * K + \frac{8}{3}$ .

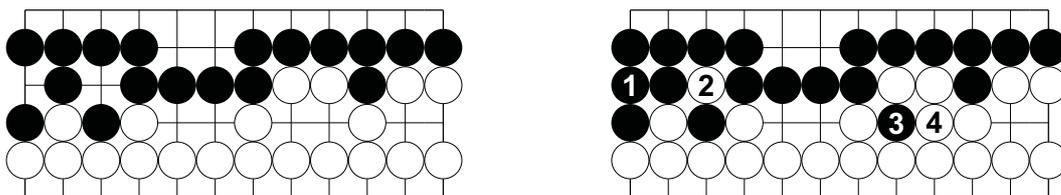
Strategy for each player: Choose the first possible option:

- 1) answer a ko threat
- 2) as the king, answer filling a dame by filling a dame
- 3) connect a ko
- 4) capture a ko
- 5) play a ko threat
- 6) as the slave, fill a dame after a ko capture

The slave can use dame instead of ko threats. The king cannot do so because then the slave would improve on the score by connecting the last ko. During the ko fight, all dame are filled.

After the king's first move, which connects a ko, the position is reduced to type 12.

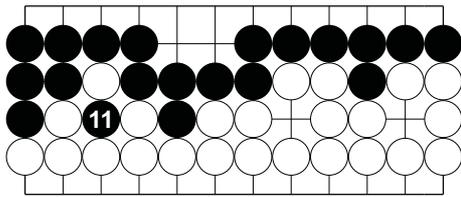
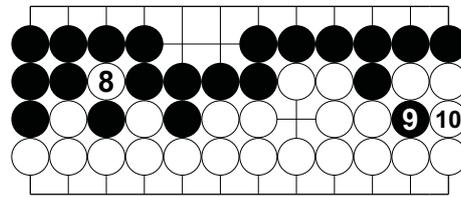
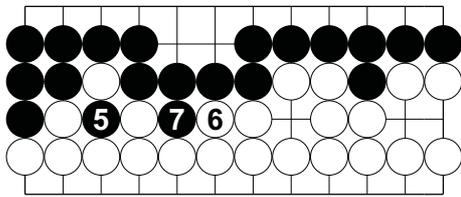
### Example



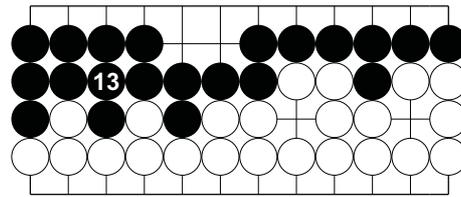
We have  $K = 2$ , the king Black,  $D = 2$ , the dame parity even,  $T = 2$ . The type's condition  $T > D/2$  is fulfilled:  $2 > 2/2$ .

Applying the formula, the score will be  $\frac{2}{3} * 2 + \frac{8}{3} = 4$ .

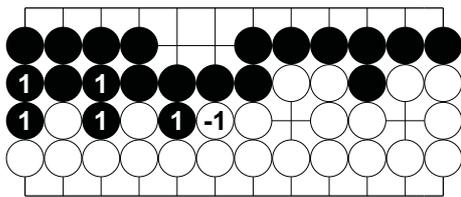
$\frac{2}{3} * 2 + \frac{8}{3} = 4$ .



12 pass.



14 pass, 15 pass.



The score is  $5 - 1 = 4$ .

## Type 21A: Remainder 2, Dame Parity Even, the King to Move, $0 < T \leq D/2$

K divided by 3 gives the remainder 2. The dame parity is even. The king moves first. The king has more ko threats but his excess of ko threats is not greater than half the number of dame.

This is an ordinarily occurring ko threat situation.

### General

There is an **ordinary ko fight** won by the **king**, but the king may choose to avoid it. **Either** player might connect the last ko.

The score is  $2/3 * K + 2/3$ .

Strategy for each player if the king chooses the ordinary ko fight: Choose the first possible option:

- 1) answer a ko threat
- 2) connect ko
- 3) capture ko
- 4) play a ko threat
- 5) fill a dame

The slave cannot force a dame ko fight because the king connects the last ko if the slave fills a dame. The king cannot force a dame ko fight about the last ko because the slave connects the last ko if the king fills a dame.

Strategy for each player if the king chooses to avoid the ordinary ko fight:

- 1) Strategy until the last ko: Choose the first possible option:
  - 1.1) connect a ko
  - 1.2) capture a ko
- 2) Strategy when only one ko remains: Choose the first possible option:
  - 2.1) as the slave's first move in (2), capture the ko
  - 2.2) connect the ko
  - 2.3) fill a dame

If one prefers a simplified view, one can use the following:

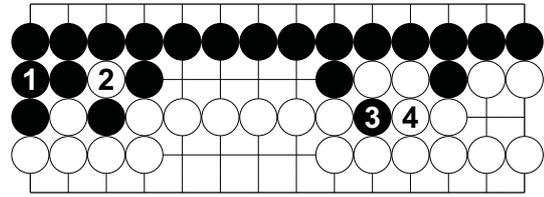
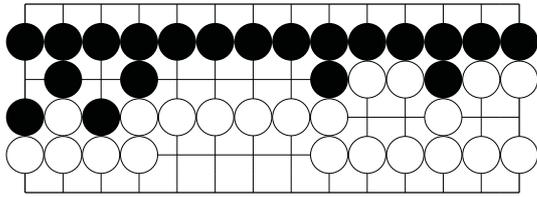
Simplified strategy for each player: Choose the first possible option:

- 1) connect a ko
- 2) capture a ko
- 3) fill a dame

After the king's first move, which connects a ko, the position is reverted to type 13A.

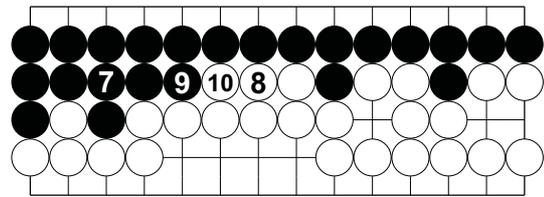
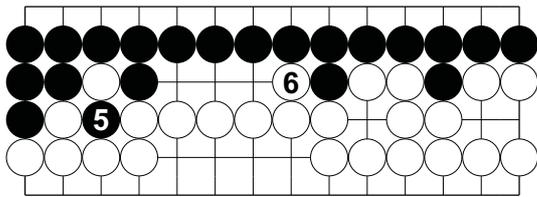
For the last ko, there are these two principle possibilities: either [the slave fills a dame - the king connects the ko - the slave fills a dame] or [the slave captures the ko - the king fills a dame - the slave connects the ko - the king fills a dame]. In both variations, the local score made on the intersections of the last ko and two involved dame is 0 and afterwards the parity of the number remaining dame is even. Therefore it does not matter which player wins the last ko with the opponent getting to play dame in the meantime, i.e., unless type 12 is created. The latest moment for the slave to start capturing the ko is when otherwise T would become greater than half the number of remaining dame. Such a type conversion would lose the slave 2 points.

## **Example**

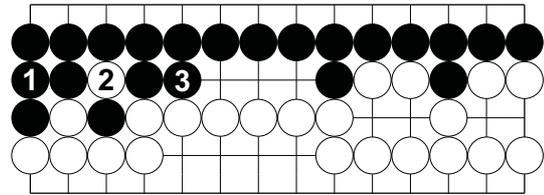
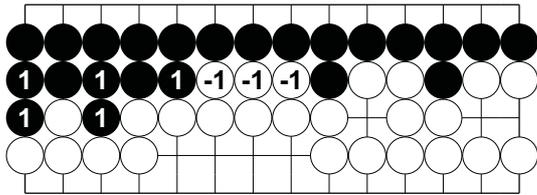


We have  $K = 2$ , the king Black,  $D = 4$ , the dame parity even,  $T = 2$ . The type's condition  $0 < T \leq D/2$  is fulfilled:  $0 < 2 \leq 4/2$ . Applying the formula, the score will be  $2/3 * 2 + 2/3 = 2$ .

**Variation 1:** Black chooses the ordinary ko fight.

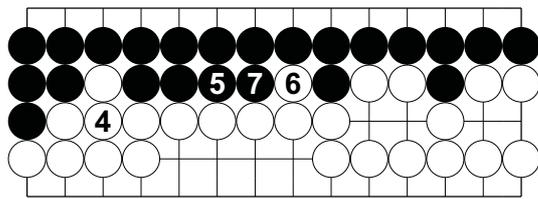


11 pass, 12 pass.

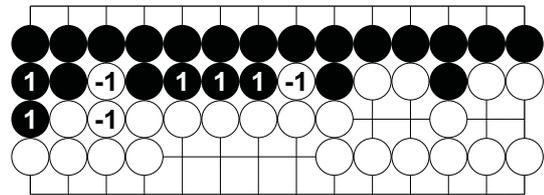


The score is  $5 - 3 = 2$ .

**Variation 2:** Black avoids the ordinary ko fight.



8 pass, 9 pass.



The score is  $5 - 3 = 2$ .

## Type 21B: Remainder 2, Dame Parity Even, the King to Move, $-D/2 < T \leq 0$

K divided by 3 gives the remainder 2. The dame parity is even. The king moves first. The slave has more or an equal number of ko threats but his excess of ko threats is not greater than or equal to half the number of dame.

This is an ordinarily occurring ko threat situation.

### General

There is an **ordinary ko fight** won by the **slave**, but the slave may choose to avoid it. **Either** player might connect the last ko.

The score is  $2/3 * K + 2/3$ .

Strategy for each player if the slave chooses the ordinary ko fight: Choose the first possible option:

- 1) answer a ko threat
- 2) connect ko
- 3) capture ko
- 4) play a ko threat
- 5) fill a dame

The king cannot force a dame ko fight about the last ko because the slave connects the last ko if the king fills a dame. The slave cannot force a dame ko fight because the king connects the last ko if the slave fills a dame.

Strategy for each player if the slave chooses to avoid the ordinary ko fight:

- 1) Strategy until the last ko: Choose the first possible option:
  - 1.1) connect a ko
  - 1.2) capture a ko
- 2) Strategy when only one ko remains: Choose the first possible option:

- 2.1) as the king's first move in (2), connect the ko
- 2.2) fill a dame

If one prefers a simplified view, one can use the following:

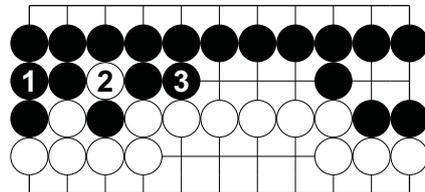
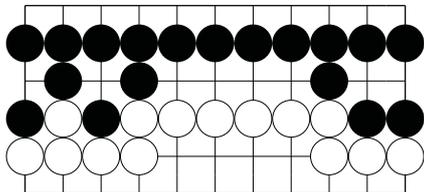
Simplified strategy for each player: Choose the first possible option:

- 1) connect a ko
- 2) capture a ko
- 3) fill a dame

After the king's first move, which connects a ko, the position is reverted to type 13B.

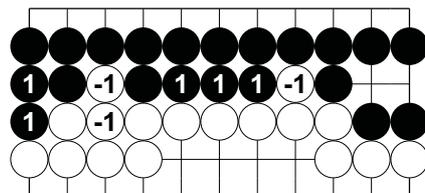
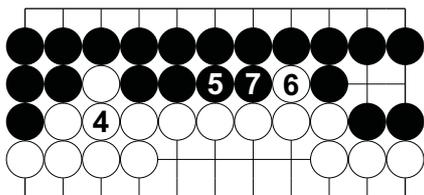
For the last ko, there are these two principle possibilities: either [the slave fills a dame - the king connects the ko - the slave fills a dame] or [the slave captures the ko - the king fills a dame - the slave connects the ko - the king fills a dame]. In both variations, the local score made on the intersections of the last ko and two involved dame is 0 and afterwards the parity of the number remaining dame is even. Therefore it does not matter which player wins the last ko with the opponent getting to play dame in the meantime, i.e., unless type 14 is created. The latest moment for the king to connect the ko is when otherwise T would become smaller than or equal to half the number of remaining dame. Such a type conversion would lose the king 2 points.

### Example



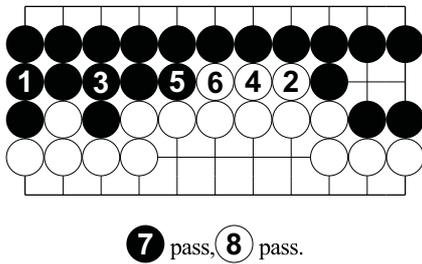
We have  $K = 2$ , the king Black,  $D = 4$ , the dame parity even,  $T = -1$ . The type's condition  $-D/2 < T \leq 0$  is fulfilled:  $-4/2 < -1 \leq 0$ . Applying the formula, the score will be  $2/3 * 2 + 2/3 = 2$ .

**Variation 1:** White chooses the ordinary ko fight. The ko fight is short because Black does not even have one ko threat; he admits defeat already at move 3.

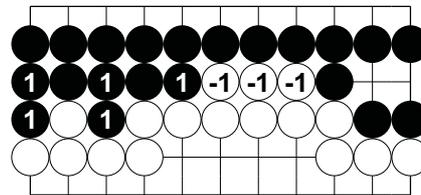


8 pass, 9 pass.

The score is  $5 - 3 = 2$ .



7 pass, 8 pass.



**Variation 2:** White avoids the ordinary ko fight.

The score is  $5 - 3 = 2$ .

## Type 22: Remainder 2, Dame Parity Even, the King to Move, $T \leq -D/2$

K divided by 3 gives the remainder 2. The dame parity is even. The king moves first. The slave has a number of ko threats that exceeds the king's number of ko threats by at least half the number of dame.

On the 19x19 board, this type occurs only occasionally.

### General

There is a **dame ko fight** won by the **slave**.

The score is  $2/3 * K - 4/3$ .

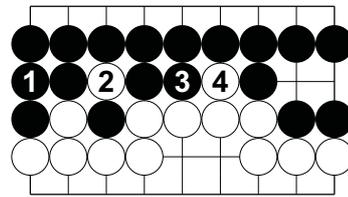
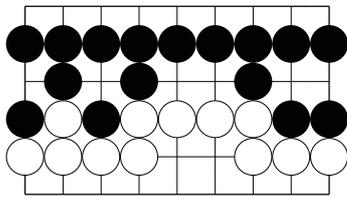
Strategy for each player: Choose the first possible option:

- 1) answer a ko threat
- 2) as the slave, answer filling a dame by filling a dame
- 3) connect a ko
- 4) capture a ko
- 5) play a ko threat
- 6) as the king, fill a dame after a ko capture

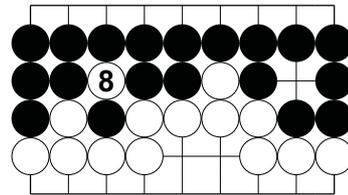
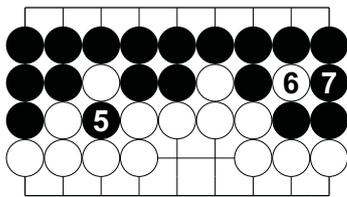
The king can use dame instead of ko threats. The slave cannot do so because then the king would improve on the score by connecting the last ko. During the ko fight, all dame are filled.

After the king's first move, which connects a ko, the position is reduced to type 14.

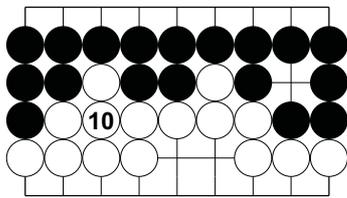
### Example



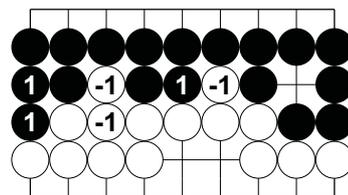
We have  $K = 2$ , the king Black,  $D = 2$ , the dame parity even,  $T = -1$ . The type's condition  $T \leq -D/2$  is fulfilled:  $-1 \leq -2/2$ . Applying the formula, the score will be  $2/3$   
 $* 2 - 4/3 = 0$ .



9 pass.



11 pass, 12 pass.



The score is  $3 - 3 = 0$ .

## Values of Types 19 to 22

The combination of types 19 + 20 means remainder 2, dame parity even, and  $T > D/2$  and has the value 4.

The combination of types 19 + 21A means remainder 2, dame parity even, and  $0 < T \leq D/2$  and has the value 2.

The combination of types 19 + 21B means remainder 2, dame parity even, and  $-D/2 < T \leq 0$  and has the value 2.

The combination of types 19 + 22 means remainder 2, dame parity even, and  $T \leq -D/2$  and has the value **0**.

## Type 23: Remainder 2, Dame Parity Odd, the Slave to Move, $T \leq -(D-1)/2$

$K$  divided by 3 gives the remainder 2. The dame parity is odd. The slave moves first. The slave has a number of ko threats that exceeds the king's number of ko threats by at least rounded down half the number of dame.

On the 19x19 board, this type occurs only occasionally.

### General

If  $D > 1$ , there is a **dame ko fight** won by the **slave**. If  $D = 1$ , it is an ordinary ko fight.

The score is  $2/3 * K - 7/3$ .

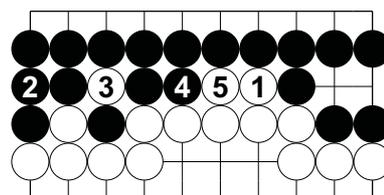
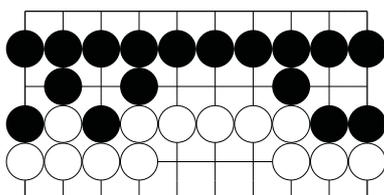
Strategy for each player: Choose the first possible option:

- 1) as the slave's first move, fill a dame
- 2) answer a ko threat
- 3) as the slave, answer filling a dame by filling a dame
- 4) connect a ko
- 5) capture a ko
- 6) play a ko threat
- 7) as the king, fill a dame after a ko capture

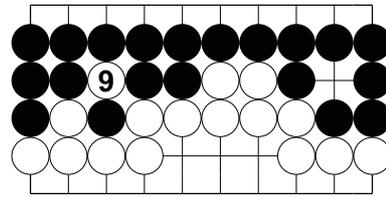
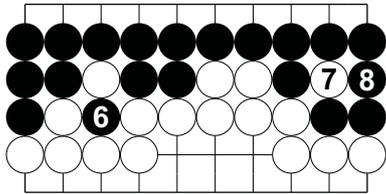
The king can use dame instead of ko threats. The slave cannot do so because then the king would improve on the score by connecting the last ko. During the ko fight, all remaining dame are filled.

After the slave's first move, which fills a dame, followed by the king's first move, which connects a ko, the position is reduced to type 14 if  $D > 1$ , else to type 4.

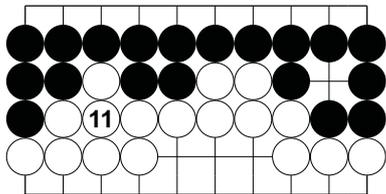
### Example



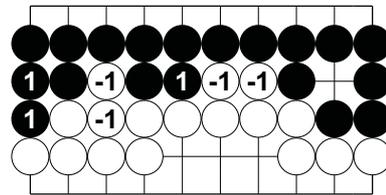
We have  $K = 2$ , the king Black,  $D = 3$ , the dame parity odd,  $T = -1$ . The type's condition  $T \leq -(D-1)/2$  is fulfilled:  $-1 \leq -(3-1)/2$ . Applying the formula, the score will be  $2/3 * 2 - 7/3 = -1$ .



10 pass.



12 pass, 13 pass.



The score is  $3 - 4 = -1$ .

## Type 24: Remainder 2, Dame Parity Odd, the Slave to Move, $T > -(D-1)/2$

$K$  divided by 3 gives the remainder 2. The dame parity is odd. The slave moves first. The slave does not have a number of ko threats that exceeds the king's number of ko threats by more than rounded down half the number of dame. It might also be the king who has more ko threats.

This is the ordinarily occurring ko threat situation. Even if the king had an excess of ko threats greater than half the number of dame, it would be useless because the slave simply avoids a ko fight by starting with filling, else capturing a ko, what leads to the remainder 0.

### General

There is **no ko fight**.

The score is  $2/3 * K - 1/3$ .

Strategy for each player: Choose the first possible option:

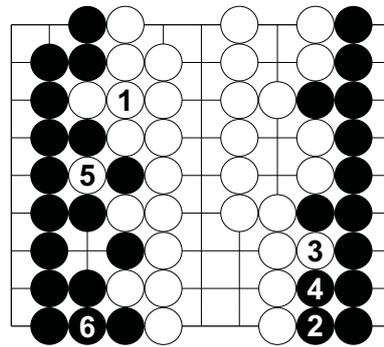
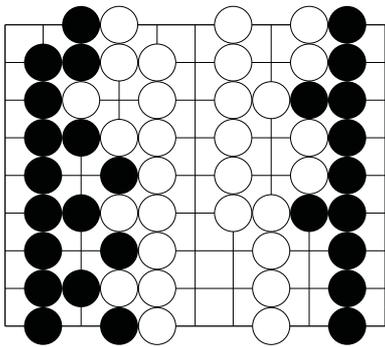
1) as the slave's first move, connect a ko, else capture a ko.

- 2) fill a dame
- 3) connect a ko
- 4) capture a ko

After the slave's first move, which connects or else captures a ko, the position is reduced to type 9 or 10.

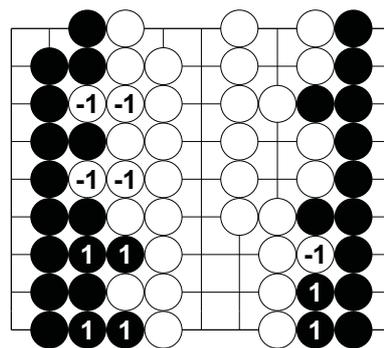
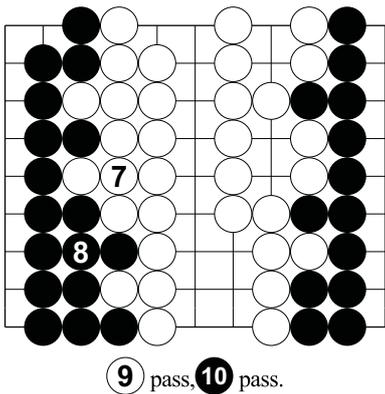
In particular as soon as only two kos remain, some variation of strategy is possible but not discussed here.

### Examples



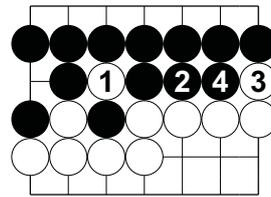
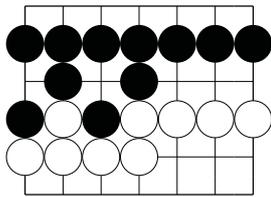
**Example A:** We have  $K = 3 - 1 = 2$ , the king Black,  $D = 3$ , the dame parity odd,  $T = 2$ . The type's condition  $T > -(D-1)/2$  is fulfilled:  $2 > -(3-1)/2$ . Applying the formula, the score will be  $2/3 * 2 - 1/3 = 1$ .

The slave's first move, connecting a ko, avoids a dame ko fight, which could occur if 1 were used to fill a dame.



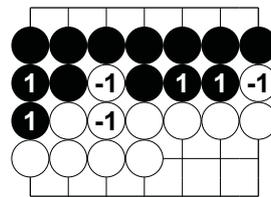
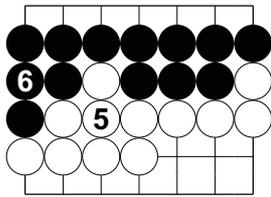
The king's excess of ko threats has been useless.

The score is  $6 - 5 = 1$ .



**Example B:** We have  $K = 2$ , the king Black,  $D = 3$ , the dame parity odd,  $T = 0$ . The type's condition  $T > -(D-1)/2$  is fulfilled:  $0 > -(3-1)/2$ . Applying the formula, the score will be  $2/3 * 2 - 1/3 = 1$ .

Here the slave's first move captures a ko. Also this leads to the remainder 0.



⑦ pass, ⑧ pass.

The score is  $4 - 3 = 1$ .

## Type 25: Remainder 2, Dame Parity Odd, the King to Move, $T > 0$

$K$  divided by 3 gives the remainder 2. The dame parity is odd. The king moves first and has more ko threats.

### General

This is an **ordinary ko fight** won by the **king**.

The score is  $2/3 * K + 5/3$ .

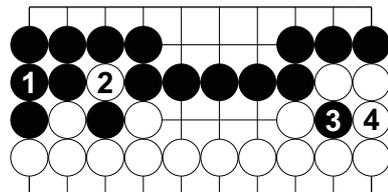
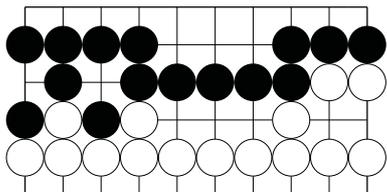
Strategy for each player: Choose the first possible option:

- 1) answer a ko threat
- 2) connect ko
- 3) capture ko
- 4) play a ko threat
- 5) fill a dame

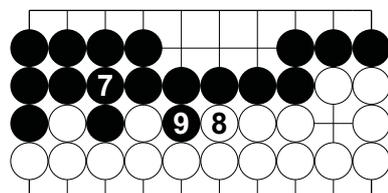
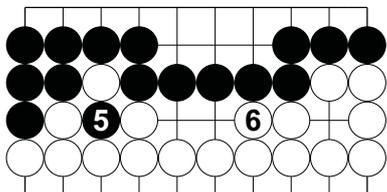
The king's first move, which connects a ko, reduces the position to type 17.

The slave cannot force a dame ko fight because the king connects the last ko as soon as the slave fills a dame. - The king cannot force a dame ko fight about the last ko because the slave can simply continue to fill dame instead of further attempting to win the ko after he has run out of ko threats. The slave loses the ko fight but this does not mean that he would also have to concede the extra odd dame to the king. In fact, the slave could fill it in his first move, admitting defeat in the ko fight before starting it at all.

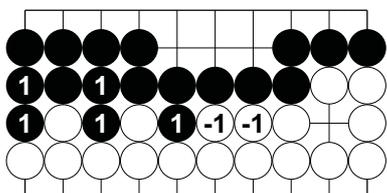
### Example



We have  $K = 2$ , the king Black,  $D = 3$ , the dame parity odd,  $T = 1$ . The type's condition  $T > 0$  is fulfilled. Applying the formula, the score will be  $2/3 * 2 + 5/3 = 3$ .



10 pass, 11 pass.



The score is  $5 - 2 = 3$ .

### Type 26: Remainder 2, Dame Parity Odd, the King to Move, $T \leq 0$

$K$  divided by 3 gives the remainder 2. The dame parity is odd. The king moves first. The slave has at least as many ko threats as the king.

## General

This is an **ordinary ko fight** won by the **slave**.

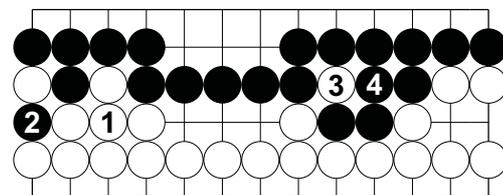
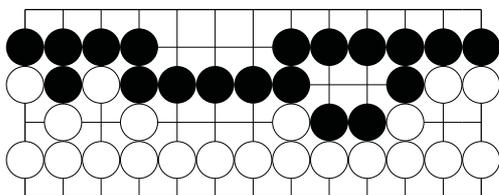
The score is  $2/3 * K - 1/3$ .

Strategy for each player: Choose the first possible option:

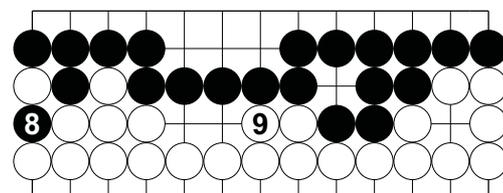
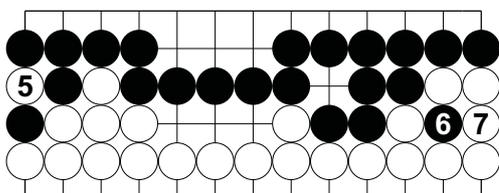
- 1) answer a ko threat
- 2) connect ko
- 3) capture ko
- 4) play a ko threat
- 5) fill a dame

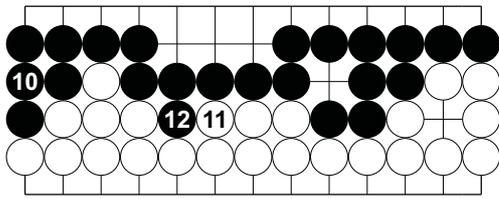
The king cannot force a dame ko fight because the slave connects the last ko as soon as the king fills a dame. - The slave cannot force a dame ko fight about the last ko because the king can simply continue to fill dame instead of further attempting to win the ko after he has run out of ko threats. The king loses the ko fight but this does not mean that he would also have to concede the extra odd dame to the slave. In fact, the king could fill it in his first move, admitting defeat in the ko fight before taking part in it at all.

## Example

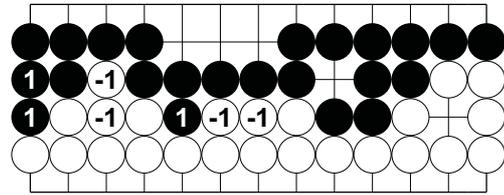


We have  $K = 2$ , the king White,  $D = 3$ , the dame parity odd,  $T = 1 - 1 = 0$ . The type's condition  $T \leq 0$  is fulfilled. Applying the formula negated for the king White, the score will be  $-(2/3 * 2 - 1/3) = -1$ .





13 pass, 14 pass.



The score is  $3 - 4 = -1$ .

## Values of Types 23 to 26

The combination of types 24 + 25 means remainder 2, dame parity odd, and  $T > 0$  and has the value **2**.

The combination of types 24 + 26 means remainder 2, dame parity odd, and  $-(D-1)/2 < T \leq 0$  and has the value **0**.

The combination of types 23 + 26 means remainder 2, dame parity odd, and  $T \leq -(D-1)/2$  and has the value **2**.

## Summary of the Types with Dame and Remainder 2

### Conditions, Ko Fights, and Score

Type	Dame Parity	To Move	T	Ko Fight	Ko Fight Winner	Black's Score - $2/3 * K$
19	even	slave	any	none		-4/3
20		king	$T > D/2$	<b>dame</b>	king	8/3
21A			$0 < T \leq D/2$	<b>ordinary</b>		slave
21B			$-D/2 < T \leq 0$			
22			$T \leq -D/2$	<b>dame</b>		-4/3
23	odd	slave	$T \leq -(D-1)/2$	<b>dame</b>	slave	-7/3
24			$T > -(D-1)/2$	none		-1/3
25		king	$T > 0$	<b>ordinary</b>	king	5/3
26			$T \leq 0$		slave	-1/3

### Values

Types	Dame Parity	T	Value
19+20	even	$T > D/2$	4
19+21A		$0 < T \leq D/2$	2
19+21B		$-D/2 < T \leq 0$	
19+22		$T \leq -D/2$	0
24+25	odd	$T > 0$	2
24+26		$-(D-1)/2 < T \leq 0$	0
23+26		$T \leq -(D-1)/2$	2

## Summary of Strategy of the Types with Dame and Remainder 1 or 2

If there is **no ko fight**, then the first move can be to connect, else capture a ko. Afterwards, strategy is peaceful but not the same in the different types. A reduction from a remainder 2 type to a simpler type is possible.

In an **ordinary ko fight**, each player's strategy is: Choose the first possible option:

- 1) answer a ko threat
- 2) connect ko
- 3) capture ko
- 4) play a ko threat
- 5) fill a dame

In a **dame ko fight**, each player's strategy is: Choose the first possible option:

- 1) If the initial dame parity is odd, fill a dame to make it even.
- 2) answer a ko threat
- 3) as the ko winner, answer filling a dame by filling a dame
- 4) connect a ko
- 5) capture a ko
- 6) play a ko threat
- 7) as the ko loser, fill a dame after a ko capture

The ko loser can use dame instead of ko threats. The ko winner cannot do so because then the ko loser would improve on the score by connecting the last ko. During the ko fight, all dame are filled.

## **Symmetry of the Types with Dame and Remainder 1 or 2**

When one compares the tables for remainder 1 with those for remainder 2, then one notices quite some analogy and similarity. However, the tables are so long that learning and applying such symmetries is not particularly easy.

# Overview on the Ko Fights

## Ordinary Ko Fights

The **size** is the difference of scores between either the king or the slave as the ko fight winner. The size 0 means that the potential ko fight winner can also choose to lose the ko fight; in this sense it is a fake.

Type	Remainder	Dame Parity	To Move	T	Ko Fight Winner	Size
3	1	none	slave	$>0$	king	4
4				$\leq 0$	slave	
6	2		king	$>0$	king	
7				$\leq 0$	slave	
13A	1	even	slave	$0 < T \leq D/2$	king	0
13B				$-D/2 < T \leq 0$	slave	
21A	2		king	$0 < T \leq D/2$	king	
21B				$-D/2 < T \leq 0$	slave	
17	1	odd	slave	$>0$	king	2
18				$\leq 0$	slave	
25	2		king	$>0$	king	
26				$\leq 0$	slave	

## Dame Ko Fights

(\*) = at the moment of the first ko connection or capture

The **size** is the difference of scores between either the dame ko fight or the no ko fight situation with another ko threat difference but otherwise the same conditions.

Type	Remainder	Dame Parity (*)	To Move (*)	T	Ko Fight Winner	Size (*)
12	1	even	slave	$T > D/2$	king	2
14				$T \leq -D/2$	slave	
15				$T > (D-1)/2$	king	
20	2		king	$T > D/2$	king	
22				$T \leq -D/2$	slave	
23				$T \leq -(D-1)/2$	slave	

## General Conclusions

The **remainder 1** corresponds with the **slave** to move while the **remainder 2** corresponds with the **king** to move.

An **even** number of dame attracts **dame ko fights**.

An **ordinary ko fight** requires (about) the **greater number** of ko threats while a **dame ko fight** requires an **excess** of ko threats that is (about) **greater than half the number of dame**.

In a dame ko fight, **only the ko loser** can use dame instead of ko threats.

# Collection of the Earlier Tables

## Conditions, Ko Fights, and Score

### Without Dame

Type	Remainder	To Move	T	Ko Fight	Ko Fight Winner	Black's Score - $\frac{2}{3} * K$
1	0	either	any	none		0
2	1	king	any	none		$\frac{4}{3}$
3		slave	$>0$	ordinary	king	$\frac{4}{3}$
4			$\leq 0$		slave	$-\frac{8}{3}$
5		slave	any	none		$-\frac{4}{3}$
6	2	king	$>0$	ordinary	king	$\frac{8}{3}$
7			$\leq 0$		slave	$-\frac{4}{3}$

### With Dame and Remainder 0

Type	Dame Parity	To Move	T	Ko Fight	Black's Score - $\frac{2}{3} * K$
8	even	either	any	none	0
9	odd	Black			1
10		White			-1

## With Dame and Remainder 1

Type	Dame Parity	To Move	T	Ko Fight	Ko Fight Winner	Black's Score - 2/3 * K
11	even	king	any	none		4/3
12		slave	$T > D/2$	<b>dame</b>	king	
13A			$0 < T \leq D/2$	<b>ordinary</b>	slave	-2/3
13B			$-D/2 < T \leq 0$			
14		$T \leq -D/2$	<b>dame</b>		-8/3	
15	odd	king	$T > (D-1)/2$	<b>dame</b>	king	7/3
16			$T \leq (D-1)/2$	none		1/3
17		slave	$T > 0$	<b>ordinary</b>	king	
18			$T \leq 0$		slave	-5/3

## With Dame and Remainder 2

Type	Dame Parity	To Move	T	Ko Fight	Ko Fight Winner	Black's Score - $2/3 * K$
19	even	slave	any	none		-4/3
20		king	$T > D/2$	<b>dame</b>	king	8/3
21A			$0 < T \leq D/2$	<b>ordinary</b>		2/3
21B			$-D/2 < T \leq 0$		slave	
22			$T \leq -D/2$	<b>dame</b>		
23	odd	slave	$T \leq -(D-1)/2$	<b>dame</b>	slave	-7/3
24			$T > -(D-1)/2$	none		-1/3
25		king	$T > 0$	<b>ordinary</b>	king	5/3
26			$T \leq 0$		slave	-1/3

## Values

### Without Dame

Types	Remainder	T	Value
1	0	any	0
2+3	1	>0	0
2+4		≤0	4
5+6	2	>0	4
5+7		≤0	0

### With Dame and Remainder 0

Types	Dame Parity	Value
8	even	0
9+10	odd	2

### With Dame and Remainder 1

Types	Dame Parity	T	Value
11+12	even	$T > D/2$	0
11+13A		$0 < T \leq D/2$	2
11+13B		$-D/2 < T \leq 0$	
11+14		$T \leq -D/2$	4
15+17	odd	$T > (D-1)/2$	2
16+17		$0 < T \leq (D-1)/2$	0
16+18		$T \leq 0$	2

### With Dame and Remainder 2

Types	Dame Parity	T	Value
19+20	even	$T > D/2$	4
19+21A		$0 < T \leq D/2$	2
19+21B		$-D/2 < T \leq 0$	
19+22		$T \leq -D/2$	0
24+25	odd	$T > 0$	2
24+26		$-(D-1)/2 < T \leq 0$	0
23+26		$T \leq -(D-1)/2$	2