1 Introduction

1.1 General

Strategy, tactical reading and evaluation are the major fields of go theory. Evaluation can be positional judgement or endgame calculation. This book explains the basics of endgame calculation.

Most endgame calculations are easy. The difficulty is knowing which are the right calculations. Our tools are values, their comparison and interpretation.

We calculate the 'counts' of positions, 'move values' of moves and other values. Despite local reading, they enable globally correct decision-making and timing of playing elsewhere.

If many moves are possible and tactical reading becomes too complex, complete reading is impossible. We only use local reading to determine local endgame values. By combining the values, we avoid global reading. This is the great advantage of modern endgame theory. It is very good at revealing the correct next moves, with only infrequent exceptions.

If we do not share the attitude of professional players who know the move values of many standard shapes by heart, we must invest more calculation effort during our games. Not only is value calculation much easier than arbitrarily complex tactical reading, but the effort invested in the endgame is very rewarding.

The endgame phase lasts ca. 150 moves, or 75 moves per player. If, on average, a player loses 1 point per move, he loses 75 points equalling several ranks. Kyus also make large mistakes. Although the endgame is a dry topic, and all endgame decisions have a similar appearance of choosing values and playing moves of certain values in a correct order, understanding endgame values well has the potential of improving one's play by several ranks. Kyu players do not notice how many points they lose during the endgame because their opponents lose a similar number of points. Endgame value theory is hardly ever taught both systematically and applicably, but if it is (as in these books) players can

improve very much by learning the theory and investing effort into applying it.

The earlier endgame literature has over-emphasised tactical reading and tesujis. Endgame evaluation has often been neglected or treated carelessly using approximations and weak explanations. This is the opposite of what we need. Over 97% of all endgame moves are decided by comparing their values while under 3% require tactical reading or tesujis, if we do not count reading the same local position more than once and exclude very easy life and death problems. The theory of this book is extraordinarily useful because it applies to almost all endgame moves.

We explain the theory and provide examples. Principles that are generally applicable are written in bold font. Some of the principles describe formulas. We often explain theory by text, formulas and examples. The examples give rise to values to be inserted in the formulas. In this way readers may choose their preferred method of understanding the theory and its application. Examples alone do not represent general application. A formula applies to all examples that fit the theory. Therefore, it is important to learn value calculations, use the right values and perform the right operations on them. The consistent naming of the variables reminds us which values we are manipulating.

The book only has simple examples so that learning the value theory is as easy as possible. If a chapter starts with difficult theory and the reader wishes to see examples, he can jump to them before proceeding with the theory. Similar life and death shapes can behave very differently, as can positions with similar move values.

This book does not give excessively short explanations, given without careful reasoning, as if the principles expounded were magic. As often as possible, we provide as much explanation as is necessary to enable the reader to make decisions that always rely on correct theory.

Most of the go theory in this book (written in bold font or in tables) relies on mathematics. The value definitions have their counter-parts in both. Such principles or formulas of this go theory rely on proved mathematical theorems, are their implications or can be proved easily with additional research time. Those theorems appear in the literature on combinatorial game theory (in particular, the book *Mathematical Go*

Endgames) or are proved by the author and appear in a later volume. Theorems are absolutely correct truths. They only have two limitations: their assumptions must be satisfied; faster methods might be discovered. Unlike combinatorial endgame theory, the go theory in this book is highly applicable. Evaluation just requires basic arithmetic and interpretation.

The reader should pass the test at the end of the book. In order to do so, he must acquire a firm understanding of the basic theory of endgame evaluation. Most tools of evaluation are easy, some have an intermediate difficulty, and the problems of the test are easy. Nevertheless, without understanding we confuse different tools and cannot pass the test successfully.

1.2 Key Concepts

In this book, we study individual local endgame positions. A **local endgame** is a local part of the board in which endgame plays are available. Usually, the considered local part is connected. During the late endgame, all or most boundaries of a local endgame are surrounded by the players' living stones.

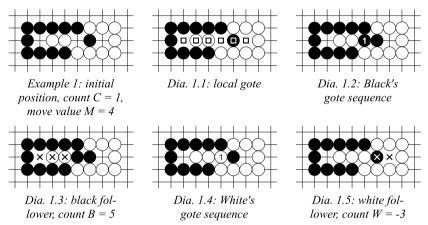
A **gote sequence** is an alternating sequence with an odd number of plays so that the same player starts and ends it. A **sente sequence** is an alternating sequence with an even number of plays so that a player starts and the opponent ends it. We use these terms regardless of types of local endgames in which the sequences occur.

There are several types of local endgame positions, of which the following two are the most frequent. A **local gote** is a local endgame with each player's gote sequence. A player's **local sente** is a local endgame with his sente sequence and the opponent's gote sequence. See *Volume 3* for precise definitions relying on value conditions.

In a local endgame, a **follower** is a follow-up position. In a local endgame, the sequence started by Black results in the *black follower* and the sequence started by White results in the *white follower*:

The **count** is the value of a position predicting the score. We consider the count **C** of the initial position of a local endgame, the count **B** of its black follower and the count **W** of its white follower. If a follower is settled, its count is Black's local points minus White's local points.

The count C of an unsettled initial position is derived from the counts of its followers. The count C of a local gote is the average of the counts B and W of its followers: the count of a local gote is C = (B + W) / 2. The count C of a local sente is the inherited count of its sente follower: Black's local sente has the count C = B inherited from the count B of the black sente follower; White's local sente has the count C = W inherited from the count W of white sente follower.

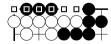


Examples 1 - 3: In the initial position, we have a local endgame. The square labels denote the local part of the board, where the local endgame is situated.

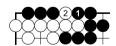
Example 1: The sequence in Dia. 1.2 is Black's gote sequence, which consists of one play so has an odd number of plays. He starts and ends the sequence. The sequence in Dia. 1.4 is White's gote sequence, which consists of one play so has an odd number of plays. He starts and ends the sequence. The local endgame is a 'local gote' because each player has a gote sequence. Black's gote sequence results in the settled black follower in Dia. 1.3 with the count B = 5. White's gote sequence results in the settled white follower in Dia. 1.5 with the count W = -3. This value is negative because we subtract White's local points in a settled position. The count C of the local gote in the initial position is the average of the counts C and C of the local gote in the initial position is the average of the counts C and C of its followers: we have C = (B + W) / 2 = (5 + (-3)) / 2 = (5 - 3) / 2 = 2/2 = 1.



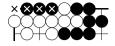
Example 2: initial position, count C = -7, move value M = 1



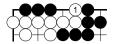
Dia. 2.1: local sente



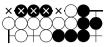
Dia. 2.2: Black's sente sequence



Dia. 2.3: black sente follower, count B = -7



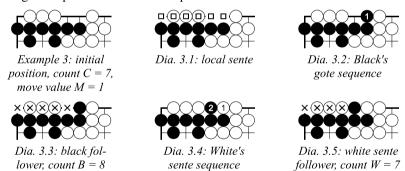
Dia. 2.4: White's gote sequence



Dia. 2.5: white follower, count W = -8

Example 2: The sequence in Dia. 2.2 is Black's sente sequence, which consists of two plays so has an even number of plays. He starts but his opponent ends the sequence. The sequence in Dia. 2.4 is White's gote sequence, which consists of one play so has an odd number of plays. He starts and ends the sequence. The local endgame is Black's 'local sente' because Black has a sente sequence and White has a gote sequence. Black's sente sequence results in the settled black sente follower in Dia. 2.3 with the count B = -7. This value is negative because we subtract White's local points in a settled position. White's gote sequence results in the settled white follower in Dia. 2.5 with the count W = -8. Again, we have a negative value. The count C of the local sente in the initial position is inherited from the count C of the black sente follower: C = B = -7.

A local sente permits one player to play his sente sequence or the opponent to play his gote sequence. We can also call the latter his 'reverse sente sequence'. White plays move 1 in *Dia. 2.4* in reverse sente. However, we do not need the concept of 'reverse sente' for identifying a local sente. It is sufficient to distinguish gote sequences from sente sequences.



Example 3: The sequence in Dia. 3.2 is Black's gote sequence, which consists of one play so has an odd number of plays. He starts and ends the sequence. The sequence in Dia. 3.4 is White's sente sequence, which consists of two plays so has an even number of plays. He starts but his opponent ends the sequence. The local endgame is White's 'local sente' because White has a sente sequence and Black has a gote sequence. Black's gote sequence results in the settled black follower in Dia. 3.3 with the count B = 8. White's sente sequence results in the settled white sente follower in Dia. 3.5 with the count W = 7. The count C of the

local sente in the initial position is inherited from the count W of the white sente follower: C = W = 7.

Traditional endgame theory was invented before modern endgame theory. Under traditional endgame theory, we multiply a sente move value by 2 to compare it with a gote move value. Under modern endgame theory, we do the converse: we divide by 2 when calculating a gote move value. As a result, each move has a value calibrated per excess play. Modern endgame theory is consistent and very powerful.

When calculating a move value M, first we calculate the difference value of the counts B and W of the followers. From the count B of the black follower, we subtract the count W of the white follower. The difference value is the **move value of a local sente:** M = B - W. Under modern endgame theory, we need to divide by 2 as the second calculation step to calculate the **move value of a local gote:** M = (B - W) / 2.

Example 1: The local gote has the count B = 5 of the black follower and count W = -3 of the white follower. Therefore, the local gote has the move value M = (B - W) / 2 = (5 - (-3)) / 2 = (5 + 3) / 2 = 8/2 = 4. We divide by 2 for a local gote.

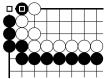
We can interpret the count C=1 and move value M=4 of the local gote in the initial position. Starting from the initial count C=1, Black gains 4 points by playing move 1 in *Dia. 1.2* and creates the black follower in *Dia. 1.3* with the count B=1+4=5. Black loses 4 points if White plays move 1 in *Dia. 1.4* and creates the white follower in *Dia. 1.5* with the count W=1-4=-3.

Example 2: The local sente has the count B = -7 of the black sente follower and count W = -8 of the white follower. Therefore, the local sente has the move value M = B - W = -7 - (-8) = -7 + 8 = 1. The move value of a local sente is simply the difference value.

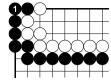
We can interpret the count C = -7 and move value M = 1 of Black's local sente in the initial position. Starting from the initial count C = -7, White causes Black to lose 1 point by playing move 1 in *Dia. 2.4* and creates the white follower in *Dia. 2.5* with the count W = -7 - 1 = -8. In Black's local sente, the move value expresses White's gain by playing in reverse sente. White's gain is Black's loss.

Example 3: The local sente has the count B=8 of the black follower and the count W=7 of the white sente follower. Therefore, the local sente has the move value M=B-W=8-7=1.

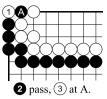
We can interpret the count C = 7 and move value M = 1 of White's local sente in the initial position. Starting from the initial count C = 7, Black gains 1 point by playing move 1 in *Dia. 3.2* and creates the black follower in *Dia. 3.3* with the







Dia. 4.1: Black starts



Dia. 4.2: White starts

Example 4: This is a basic endgame ko, whose fight is about the ko stone.

Dia. 4.1: The count of the black follower is B = 0.

Dia. 4.2: White captures and connects the ko so he has 1 prisoner. The count of the white follower is W = -1.

Example 4 conclusion: The difference value is B - W = 0 - (-1) = 1.

2.10 Local versus Global Gote or Sente

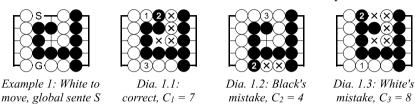
There are local gote and global gote, and local sente and global sente. See *Volume 3* for precise definitions of the local types by value conditions. Informally, a *local gote* is a local endgame without a sente sequence, a player's *local sente* is a local endgame with his sente sequence and an *ambiguous* local endgame is a hybrid of local gote and local sente. A *global sente* is a local endgame whose sente sequence is currently correct play for both players. A *global gote* is a local endgame in which a gote sequence is currently correct play.

Usually, we play a local gote when it is a global gote, and we play a local sente when it is a global sente. However, in some exceptional cases, a local gote can be a global sente, a local sente can be a global gote and a player's local sente can be the opponent's global sente.

In addition, there are these types of global sentes or related global gotes: a local sente that is also currently a global sente; a local sente for which currently the best global play is the local sente sequence continued by a local gote or reverse sente follow-up; a local sente played in reserve sente whose local gote, sente or reverse sente follow-up is the best global play; a local gote whose local gote or reverse sente follow-up is the best global play.

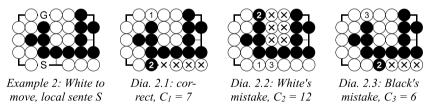
Whether a local endgame is a global sente or global gote depends on move values in the *environment*, which consists of the other local endgames elsewhere on the board, and other strategic aspects. Normally,

at some point, we play a local gote as a global gote, and we play a local sente as a global sente. However, there are exceptions. If a local gote becomes a global sente, the player's local gote play is followed by the opponent's local reply as it is his best play on the whole board. A local sente is a global gote if, for example, the opponent has a local gote in the environment that is more valuable than the local follow-up.

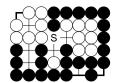


All examples: Every label denotes a local endgame on the intersection of its first play. The region of the local endgame consists of all intersections surrounded by the defender's stones.

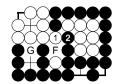
Example 1: The local endgame S is White's local sente and a global sente. Currently, it is correct for both players to play the sente sequence White 1 - 2 in Dia. 1.1 before White 3 in the local gote G. The resulting count is $C_1 = 7$. On move 2, Black avoids the mistake 2 in Dia. 1.2 resulting in the smaller count $C_2 = 4$. On move 1, White avoids the mistake 1 in Dia. 1.3 resulting in the larger count $C_3 = 8$. In the initial position, the local endgame S is a global sente because Black's local reply 2 in Dia. 1.1 is more valuable than his play 2 elsewhere in Dia. 1.2.



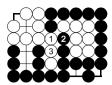
Example 2: The local endgame S is White's local sente but not a global sente. Currently, the correct moves are White 1 in $Dia.\ 2.1$ in the local gote G and the reverse sente Black 2 in White's local sente. The resulting count is $C_1 = 7$. On move 1, White avoids the mistake 1 in $Dia.\ 2.2$ resulting in the larger count $C_2 = 12$. On move 2, Black avoids the mistake 2 in $Dia.\ 2.3$ resulting in the smaller count $C_3 = 6$. Although White 1 in $Dia.\ 2.2$ is played in the local sente, it is not a global sente because Black's reply 2 elsewhere is more valuable than his local reply in $Dia.\ 2.3$.



Example 3: White to move, local sente + global gote

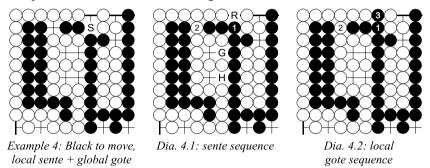


Dia. 3.1: sente sequence

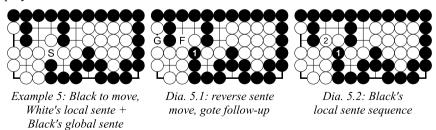


Dia. 3.2: local gote sequence

Example 3: The local endgame S is White's local sente and a global gote because the currently best global play is the sente sequence White 1 - 2 in *Dia. 3.1* + 3.2 continued with the local gote follow-up F (move 3 in *Dia. 3.2*). This follow-up is more valuable than the local gote G elsewhere.

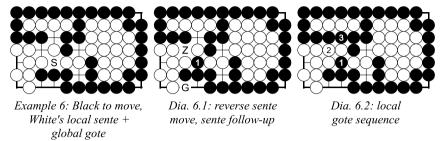


Example 4: In the initial position, the local endgame S is Black's local sente with White's sente follow-up. In the position created by move 2, the local endgame R is White's local sente and a global sente. However, White never gets a chance to play it. In the initial position, Black's local sente is a global gote because the currently best global play is the sente sequence Black 1 - 2 in Dia. 4.1 + 4.2 continued with Black's reverse sente follow-up R (move 3 in Dia. 4.2). This follow-up is more valuable than the local gotes G and H elsewhere. If the local gote H were slightly more valuable than the follow-up R, Black would play move 3 in H. White 2 at 3 would be a mistake.

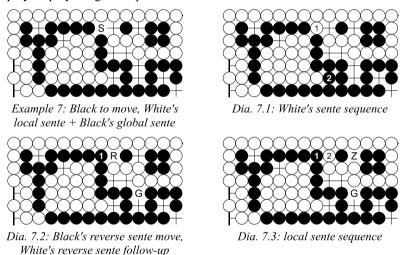


Example 5: In the initial position, the local endgame S is White's local sente whose black reverse sente follower created by Black 1 has the gote follow-up F.

Nevertheless, the players play the local endgame S as a global sente because F is more valuable than the local gote G elsewhere. The local endgame S is White's local sente but Black's global sente.



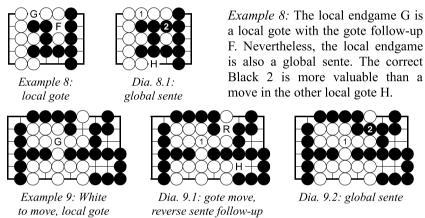
Example 6: The local endgame S in the initial position is White's local sente whose black reverse sente follower created by Black 1 is White's local sente Z in *Dia. 6.1*. The follow-up is more valuable than the local gote G elsewhere. Therefore, the local endgame S in the initial position is a global gote, in which the players play the gote sequence in *Dia. 6.2*.



Example 7: The local endgame S in the initial position is White's local sente with the sente sequence in Dia. 7.1. After Black's reverse sente move 1 in Dia. 7.2, White has the reverse sente move R. Since this is more valuable than a move in the local gote G elsewhere, the players play sente sequence in Dia. 7.3. Therefore, the local endgame S in the initial position is White's local sente and Black's global sente. If the local gote G were slightly more valuable than White 2 in Dia. 7.3, the local endgame S in the initial position would not be a global

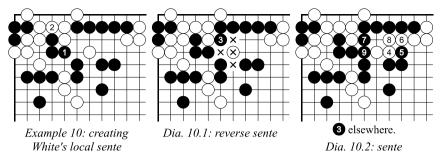
sente. Instead, the players would play the sequence Black 1 - G - 2 - Z in *Dia*. 7.3 so that the local evaluation of the local endgame S in the initial position as White's local sente would describe the ordinary behaviour of White's local sente sequence (*Dia*. 7.1) or Black's local reverse sente move 1 (*Dia*. 7.2).

It is an exception that the local endgame S is Black's global sente. The exception is caused by the absence of other local endgames elsewhere whose moves would be more valuable than a move at R. Needless to say, a player's reverse sente play followed by the opponent's local reverse sente reply (like in *Dia. 7.3*) is scarce. Since White 2 in *Dia. 7.3* is less valuable than Black 1, the local endgame S in the initial position is not a local double sente. Although later volumes study non-existence of local double sentes, there are global double sentes, such as the local endgame S in the initial position considered with either starting player.

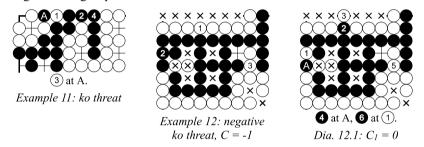


Example 9: In the initial position, the local endgame G is a local gote but global sente. After White 1, Black's reverse sente move R (Black 2) is more valuable than Black H is the other local gote. Therefore, the players play the sente sequence in *Dia. 9.2*.

There are also the following exceptions: a recently created local sente is played as a reverse sente because the reverse sente player can do so before the sente player gets his first chance of playing his local sente; a local sente with sente and gote options (see *Volume 3*) is played in gote because it is the last local endgame of the game; a ko threat or negative ko threat is played in the local endgame. See later volumes for further exceptions caused by unusual values in the environment.



Example 10: Black 1 is sente against the life of the white corner group and creates White's local sente in the upper center. Before White can play it as in *Dia.* 10.2, Black plays reverse sente in *Dia.* 10.1, gains the marked 5 points and strengthens his group.



Example 11: Black 2 is a ko threat played in a local sente. White has to ignore the ko threat so that his larger corner group survives.

Example 12: White wins by playing the negative ko threat 1, which eliminates black ko threats. Although Black wins the ko, White wins the game with the final count C = -1. Move 1 is a reverse sente with the smallest available move value 1 played when the ko move value 7/3 is much larger and the remaining simple gote has the slightly larger move value 1.5. (For the sake of simplicity, we evaluate the ko move value like for an ordinary ko. See 3.1.4 Move Value (p. 55) for an explanation of move values.) If Black plays 2 at 3, White wins the ko because Black has no ko threat. In Dia. 12.1, White makes the mistake to fight the ko, only achieve a tie with the count $C_1 = 0$ and allow Black to play his local sente. Locally, this is expected. Globally, White trusts ordinary behaviour and evaluation too much. Ko fights require additional analysis.

2.11 Played versus Delayed Local Sente Sequences

Usually during the early endgame, a player has a period of time during which he can play the sente sequence of a local sente of his when it is

3.1.1 Introduction to Counts

For middle game evaluation, 'territorial positional judgement' (see the book *Positional Judgement 1 - Territory*) is the preferred means for assessing territory. For endgame evaluation, more exact means are needed. If we neither use the method of reading and counting nor full-fledged combinatorial game theory, our means is determination of the 'count' of every local endgame. We assess a position or local endgame by its count.

The count of a local gote is calculated as an average of the counts of the black and white followers. The count of a local sente is the inherited count of the sente follower. Kos and other types of local positions use other calculations of the count.

If the local endgames are independent of each other, the count of the whole board position is the sum of the counts of its local endgames adjusted by the komi. To calculate the count, we temporarily ignore whose turn it is to move. Instead, we consider these two cases: Black starts local play versus White starts local play. Furthermore, we make the simplifying assumption that other strategic concepts do not play a role: changing life and death statuses of surrounding groups; exploitation of shape weaknesses ('aji'); influence; ko threats and so on. See the literature for their impact.

3.1.2 Count of a Local Gote

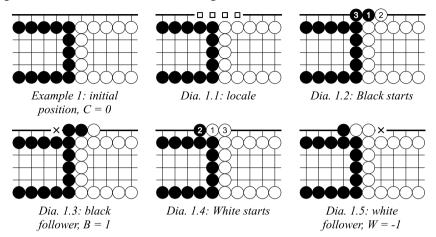
The count C of a local gote is the average of the counts B and W of its followers:

$$C = (B + W) / 2$$

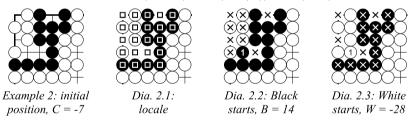
We have two numbers, which are the count B of the black follower and the count W of the white follower, and calculate their average. According to 7.2.6 Average (p. 248), the average of two numbers is their sum divided by two. This is done in the formula. All counts are locally restricted to the locale.

A gote count is the average of its followers because it expresses the estimated, expected value, which assumes a 50% likelihood of the result becoming the count of the black follower and a 50% likelihood of the result becoming the count of the white follower. As we learn in 3.1.4

Move Value (p. 55), the formula for a gote move value looks very similar to the formula for a gote count. We avoid confusion by recalling that a gote count is calculated as an average.



Example 1: The local endgame is a local gote with Black's gote sequence in Dia. 1.2 and White's gote sequence in Dia. 1.4. The count of the black follower in Dia. 1.3 is B = 1. The count of the white follower in Dia. 1.5 is W = -1. The count C of the initial position in the locale in Dia. 1.1 is the average of the counts of its followers: C = (B + W) / 2 = (1 + (-1)) / 2 = (1 - 1) / 2 = 0 / 2 = 0.



Example 2: The local endgame is a local gote with Black's gote sequence in Dia. 2.2 and White's gote sequence in Dia. 2.3. The count of the black follower is B = 14. The count of the white follower is W = -28. The count C of the initial position in the locale is the average of the counts of its followers: C = (B + W) / (2 + (-28))

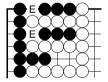
If the followers are settled local positions, we calculate their counts directly. If a follower is unsettled, we iterate its calculation by first determining the count of the follower from the counts of its own followers. We might need to iterate for both followers of the local gote of the initial position. See 3.2 Local Endgames with Follow-ups (p. 86).

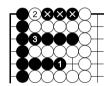
5.4.1 Ignoring Equal Options

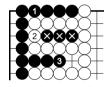
First ignore equal options.

Also during the microendgame, we can ignore equal options. In fact, we sometimes have to ignore them. In microendgame positions with those basic shapes we study, this principle has the highest priority preceding the other principles for move order during the microendgame. Ignoring equal options simplifies the decision-making and enables some correct decisions. See also *Examples 8 - 10* in *5.3.1 Empty Corridors* (p. 139). The two local endgames paired as equal options need not have the same shape but must have the same move value and move-related structure. Furthermore, we restrict ourselves to corridors whose gold is settled by the one move on the last empty intersection of the corridor's head. Later, when only the equal options remain, play continues in them. The two basic kinds of equal options are as follows:

- two simple gotes with the same move value,
- two colour-reversed corridors.







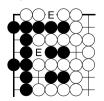
Example 1: Black to move

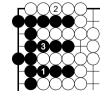
Dia. 1.1: correct I, $C_1 = -6$ Dia. 1.2: correct II, $C_2 = -6$

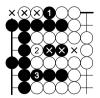
All examples: Equal letters denote equal options, which can be ignored on move 1, as in the first variation.

Example 1: The two simple gotes are equal options because they have the same move value 3. In *Dia. 1.1*, Black applies the principle and can start with the smaller simple gote.

Example 2: Simple gotes paired in equal options can have the same colour (Example 1) or reversed colours (Example 2).



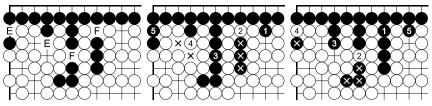




Example 2: Black to move

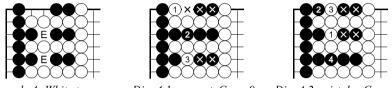
Dia. 2.1: correct I, $C_1 = 0$

Dia. 2.2: correct II, $C_2 = 0$



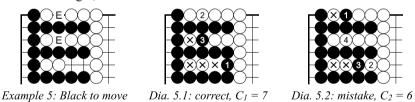
Example 3: Black to move Dia. 3.1: correct I, $C_1 = -8$ Dia. 3.2: correct II, $C_2 = -8$

Example 3: Regardless of the different shapes, we identify two pairs of equal options. Each pair has the same move value and move-related structure: after one move in a local endgame, there is no further local valuable move.

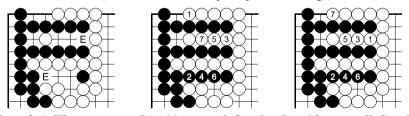


Example 4: White to move Dia. 4.1: correct, $C_1 = -9$ Dia. 4.2: mistake, $C_2 = -8$

Example 4: White has to ignore the marked equal options as in Dia. 4.1. Although the reverse sente White 1 has the move value 1 and White's move at 2 or 3 would have the move value 2, his correct start results in the smaller, more favourable count $C_1 = -9$. If instead he makes the mistake 1 in Dia. 4.2, he achieves the larger, less favourable count $C_2 = -8$.



Example 5: Black 1 correctly ignores the equal options and achieve the larger count $C_1 = 7$ (Dia. 5.1). His start with an equal option is wrong (Dia. 5.2).



Example 6: White to move Dia. 6.1: correct I, $C_1 = 0$ Dia. 6.2: correct II, $C_2 = 0$

Example 6: In this example with two rich corridors being equal options, both strategies are correct - to ignore or attack them.