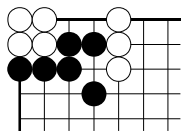


## Local Sente Endgame

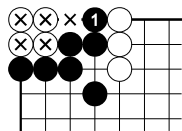
► In a local sente endgame, the *move value* is the difference of the count of Black's follow-up position and the count of White's follow-up position, where one is created by a sente sequence and the other is created in reverse sente.

► In a local sente endgame, the *count* is inherited from the count of the follow-up position created by the sente sequence.

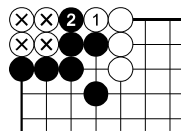
► In a local sente endgame, the move value of the initial position is smaller than the follow-up move value during the sente sequence.



*Dia. 1:* Move value = 1, count = 8

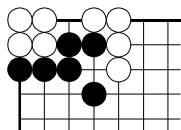


*Dia. 2:* Black starts: Count = 9

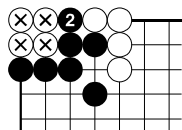


*Dia. 3:* White starts: Count = 8

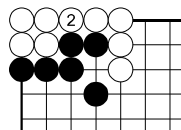
The reverse sente move 1 in *Dia. 2* creates Black's follow-up position with the count 9 (four occupied intersections and one empty intersection). The sente sequence in *Dia. 3* creates White's follow-up position with the count 8 (four occupied intersections). In the initial position of *Dia. 1*, the local sente endgame's sente *move value* is the difference of the count 9 of Black's follow-up position and the count 8 of White's follow-up position:  $9 - 8 = 1$ . The initial position's *count* 8 is inherited from the count 8 of White's follow-up position created by the sente sequence, which we expect to be played sooner or later.



*Dia. 4:* Follow-up move value = 4, count = 4



*Dia. 5:* Black continues: Count = 8



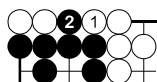
*Dia. 6:* White continues: Count = 0

White 1 in *Dia. 3* creates the intermediate position in *Dia. 4*. At this moment, the local endgame is a local gote endgame. We calculate its gote move value, which is the *follow-up move value*, as half the difference of the count 8 of the position created by Black's continuation in *Dia. 5* and the count 0 of the position created by White's continuation in *Dia. 6* so  $(8 - 0) / 2 = 4$ . For the local sente endgame in *Dia. 1*, the initial position's move value 1 is smaller than the follow-up move value 4, that is,  $1 < 4$ . Note that, in *Dia. 4*, the intermediate position's count is the average of the count 8 of the position created by Black's continuation and the count 0 of the position created by White's continuation so  $(8 + 0) / 2 = 4$ .

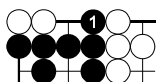
## Shortcut IV for a Local Sente

► In a local sente endgame, compare different territory intersections of the positions created by the sente and reverse sente sequences. The sum of the following points is the sente move value.

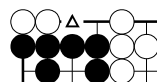
- Each such settled initially occupied intersection contributes 2 points.
- Each such settled initially empty intersection contributes 1 point.
- Each occupied intersection of simple gote follow-ups contributes 1 point.
- Each empty intersection of simple gote follow-ups contributes 1/2 point.



Sente sequence

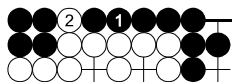


Reverse sente sequence

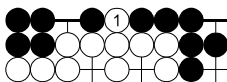


Move value = 1

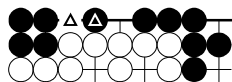
*Example 1:* The one marked empty intersection contributes 1 point. This is the move value.



Sente sequence

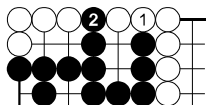


Reverse sente sequence

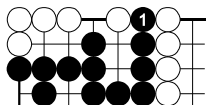


Move value = 3

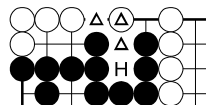
*Example 2:* The one marked occupied intersection contributes 2 points and the one marked empty intersection adds 1 point. The sum 3 is the move value.



Sente sequence

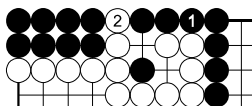


Reverse sente sequence

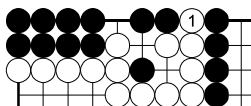


Move value = 4 1/2

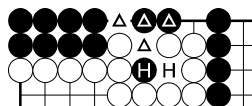
*Example 3:* The one marked occupied intersection contributes 2 points. Each of the two empty intersections marked with a triangle contributes 1 point. The empty half territory H of the simple gote follow-up contributes 1/2 point. The sum  $2 + 2 * 1 + 1/2 = 4 1/2$  is the move value.



Sente sequence



Reverse sente sequence

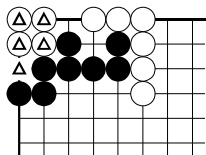


Move value = 7 1/2

*Example 4:* Each of the two occupied intersections marked with a triangle contributes 2 points. Each of the two empty intersections marked with a triangle contributes 1 point. The occupied half territory H of the simple gote follow-up contributes 1 point. The empty half territory H of the simple gote follow-up contributes 1/2 point. The sum  $2 * 2 + 2 * 1 + 1 + 1/2 = 7 1/2$  is the move value.

## Answer 4

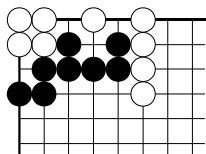
### Follow-up



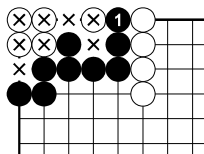
*Dia. 1:* Follow-up: Move value =  $4 \frac{1}{2}$ , count =  $4 \frac{1}{2}$

*Dia. 1:* Applying shortcut I, the four marked intersections contribute 4 and the one marked empty intersection contributes  $\frac{1}{2}$  so the *follow-up move value* is  $4 \frac{1}{2}$  and the *count* is  $4 \frac{1}{2}$ .

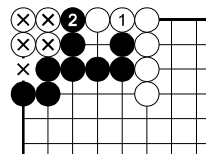
### Initial Position



*Dia. 2:* Initial position: Move value = 4, count = 9



*Dia. 3:* Black starts: Count = 13



*Dia. 4:* White's sente sequence: Count = 9

*Dia. 3:* Black has 13 points (five occupied and three empty intersections) so the count is 13. - *Dia. 4:* Black has 9 points (four occupied intersections and one empty intersection) so the count is 9. White 1 creates the follow-up in *Dia. 1*.

*Move value:* In *Dia. 2*, the initial position's *move value* is the difference of the count 13 of the follow-up position in *Dia. 3* created by Black's move and the count 9 of the follow-up position in *Dia. 4* created by White's sente sequence:

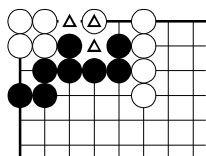
$$13 - 9 = 4.$$

*Count:* The initial *count* 9 is inherited from the count 9 of the follow-up position in *Dia. 4* created by White's sente sequence.

*Verification:* The initial position's local endgame is a local sente because the move value 4 is smaller than the follow-up move value  $4 \frac{1}{2}$ , that is,  $4 < 4 \frac{1}{2}$ .

*Interpretation:* Starting from the initial position with the count 9, the reverse sente Black 1 in *Dia. 3* gains 4 to create Black's follow-up position with the count 13. White's sente sequence in *Dia. 4* has the net profit 0.

### Shortcut



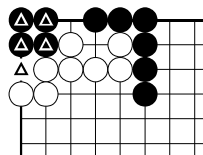
*Dia. 5:* Shortcut

*Dia. 5:* Applying shortcut IV, the one marked occupied intersection contributes 2 and the two marked empty intersections contribute 2 so the move value is 4.

## Answer 10

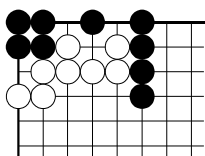
### Follow-up

*Dia. 1:* Applying shortcut I, the four marked occupied intersections contribute 4 and the one marked empty intersection contributes 1/2 so the *follow-up move value* is  $4 \frac{1}{2}$  and the *count* is  $-4 \frac{1}{2}$ , which is negative and favours White.

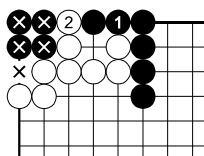


*Dia. 1:* Follow-up: Move value =  $4 \frac{1}{2}$ , count =  $-4 \frac{1}{2}$

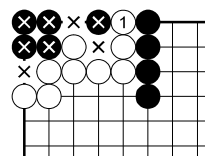
### Initial Position



*Dia. 2:* Initial position:  
Move value = 4, count = -9



*Dia. 3:* Black's sente sequence: Count = -9



*Dia. 4:* White starts:  
Count = -13

*Dia. 3:* Black has 0 points. We subtract White's 9 points (four occupied intersections and one empty intersection) so the count is -9. Black 1 creates the follow-up in *Dia. 1*. - *Dia. 4:* Black has 0 points. We subtract White's 13 points (five occupied and three empty intersections) so the count is -13. - The negative counts favour White.

*Move value:* In *Dia. 2*, the initial position's *move value* is the difference of the count -9 of the follow-up position in *Dia. 3* created by Black's sente sequence and the count -13 of the follow-up position in *Dia. 4* created by White's move:

$$-9 - (-13) = 4.$$

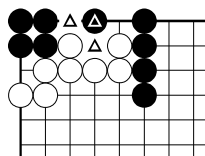
*Count:* The initial count -9, which favours White, is inherited from the count -9 of the follow-up position in *Dia. 3* created by Black's sente sequence.

*Verification:* The initial position's local endgame is a local sente because the move value 4 is smaller than the follow-up move value  $4 \frac{1}{2}$ , that is,  $4 < 4 \frac{1}{2}$ .

*Interpretation:* Black's sente sequence in *Dia. 3* has the net profit 0. Starting from the initial position with the count -9, the reverse sente White 1 in *Dia. 4* gains 4 to create White's follow-up position with the count -13.

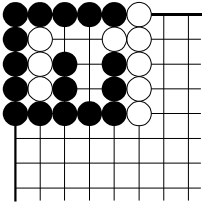
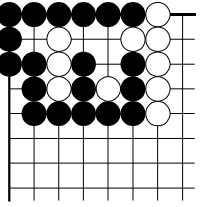
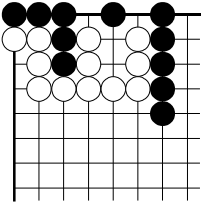
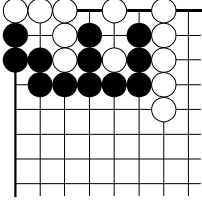
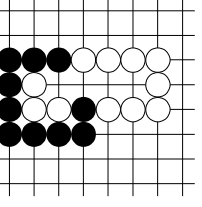
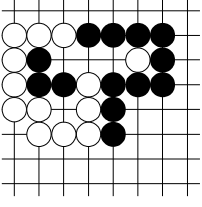
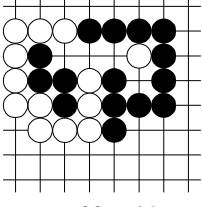
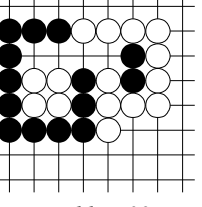
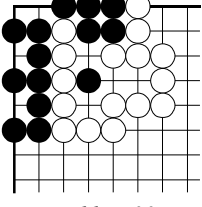
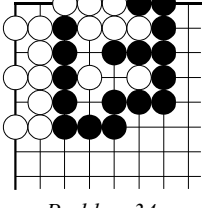
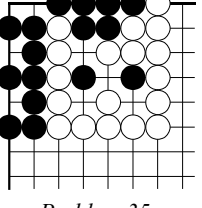
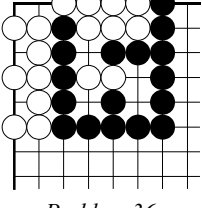
### Shortcut

*Dia. 5:* Applying shortcut IV, the one marked occupied intersection contributes 2 and the two marked empty intersections contribute 2 so the move value is 4.



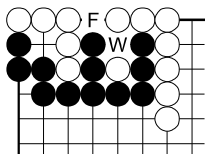
*Dia. 5:* Shortcut

### 3 Sente with Gote Follow-ups

 <p><i>Problem 25</i></p>	 <p><i>Problem 26</i></p>	 <p><i>Problem 27</i></p>
 <p><i>Problem 28</i></p>	 <p><i>Problem 29</i></p>	 <p><i>Problem 30</i></p>
 <p><i>Problem 31</i></p>	 <p><i>Problem 32</i></p>	 <p><i>Problem 33</i></p>
 <p><i>Problem 34</i></p>	 <p><i>Problem 35</i></p>	 <p><i>Problem 36</i></p>

## Answer 28

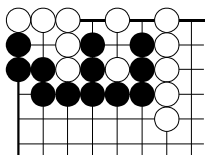
### Follow-ups



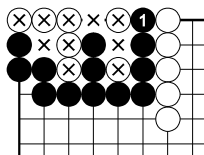
*Dia. 1:* Follow-ups:  
Move value =  $5 \frac{1}{2}$   
Count =  $6 \frac{1}{2}$

*Dia. 1:* We apply shortcut I to the simple gotes without follow-ups. In region F, the five occupied intersections contribute 5 and the one empty intersection contributes  $\frac{1}{2}$  so its move value is  $5 \frac{1}{2}$  and its count is  $5 \frac{1}{2}$ . In region W in *Dias. 1* and *4*, the one occupied intersection contributes 1 so its move value is 1 and its count is 1. In the follow-up position, the *follow-up move value*  $5 \frac{1}{2}$  is the maximum of both regions' individual move values and the *count* is the sum  $5 \frac{1}{2} + 1 = 6 \frac{1}{2}$  of both regions' individual counts.

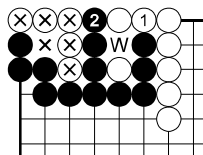
### Initial Position



*Dia. 2:* Initial position:  
Move value = 5, count = 12



*Dia. 3:* Black starts:  
Count = 17



*Dia. 4:* White's sente sequence: Count = 12

*Dia. 3:* Black has 17 points (seven occupied and three empty intersections) so the count is 17. - *Dia. 4:* Black has 11 points (five occupied intersections and one empty intersection) in the settled region and 1 point in region W. The count 12 is the sum of both regions' individual counts. Due to shortcut III, the long sequence White 1 - Black 2 - White W is not worth playing successively. White 1 creates the follow-up position in *Dia. 1*.

*Move value:* In *Dia. 2*, the initial position's *move value* is the difference of the count 17 of the follow-up position in *Dia. 3* created by Black's move and the count 12 of the follow-up position in *Dia. 4* created by White's sente sequence:

$$17 - 12 = 5.$$

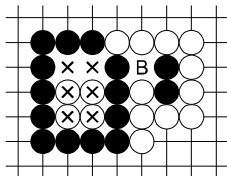
*Count:* The initial *count* 12 is inherited from the count 12 of the follow-up position in *Dia. 4* created by White's sente sequence.

*Verification:* The initial position's local endgame is a local sente as the move value 5 is smaller than the follow-up move value  $5 \frac{1}{2}$  in *Dia. 1* so  $5 < 5 \frac{1}{2}$ .

*Interpretation:* Starting from the initial position with the count 12, the reverse sente Black 1 in *Dia. 3* gains 5 to create Black's follow-up position with the count 17. White's sente sequence in *Dia. 4* has the net profit 0.

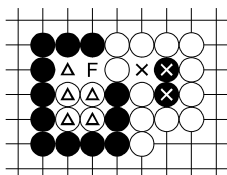
## Answer 32

### Black's Follow-up



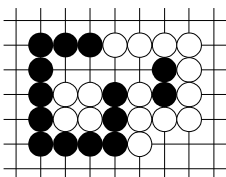
*Dia. 1:* Black's follow-up: Move value = 2  
Count = 8

### White's Follow-up

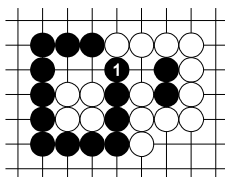


*Dia. 2:* White's follow-up: Move value = 4 1/2  
Count = -1/2

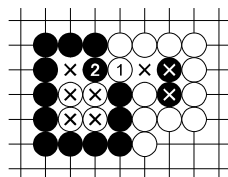
### Initial Position



*Dia. 3:* Initial position:  
Move value = 4, count = 4



*Dia. 4:* Black starts



*Dia. 5:* White's sente sequence: Count = 4

*Dia. 4:* Black 1 creates Black's follow-up in *Dia. 1* - *Dia. 5:* Black has 9 points (four occupied intersections and one empty intersection). We subtract White's 5 points (two occupied intersections and one empty intersection) so the count is 4.

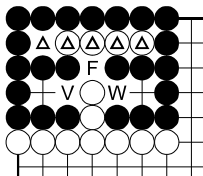
*Move value:* In *Dia. 3*, the initial position's *move value* is the difference of the count 8 of Black's follow-up position in *Dia. 1* and the count 4 of the follow-up position in *Dia. 5* created by White's sente sequence:  $8 - 4 = 4$ .

*Count:* The initial *count* 4 is inherited from the count 4 of the follow-up position in *Dia. 5* created by White's sente sequence.

*Verification:* The initial position's local endgame is a local sente: the move value 4 is smaller than White's follow-up move value 4 1/2 in *Dia. 2* so  $4 < 4 \frac{1}{2}$ .

## Answer 37

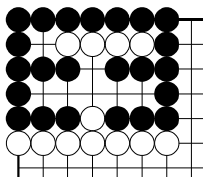
### Follow-ups



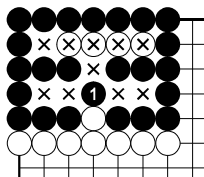
*Dia. 1:* Follow-ups:  
Move value =  $4 \frac{1}{2}$   
Count =  $5 \frac{1}{2}$

*Dia. 1:* We apply shortcut I to the simple go tes without follow-ups. In region F, the four occupied intersections contribute 4 and the one empty intersection contributes  $\frac{1}{2}$  so its move value is  $4 \frac{1}{2}$  and its count is  $4 \frac{1}{2}$ . In each of the regions V and W in *Dias. 1* and *4*, the one empty intersection contributes  $\frac{1}{2}$  so either move value is  $\frac{1}{2}$  and either count is  $\frac{1}{2}$ . In the follow-up position, the *follow-up move value*  $4 \frac{1}{2}$  is the maximum of all regions' individual move values and the *count* is the sum  $4 \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 5 \frac{1}{2}$  of all regions' individual counts.

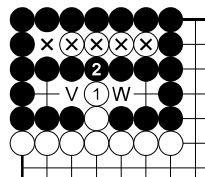
### Initial Position



*Dia. 2:* Initial position:  
Move value = 4, count = 10



*Dia. 3:* Black starts:  
Count = 14



*Dia. 4:* White's sente sequence: Count = 10

*Dia. 3:* Black has 14 points (four occupied and six empty intersections) so the count is 14.

*Dia. 4:* Black has 9 points (four occupied intersections and one empty intersection) in the settled region,  $\frac{1}{2}$  point in region V and  $\frac{1}{2}$  point in region W. The count 10 is the sum of all regions' individual counts. During the sente sequence, White 1 creates the follow-up position in *Dia. 1*.

*Assumption:* We assume that the initial position's local endgame in *Dia. 2* is a local sente and its values depend on White's follow-up position created by his sente sequence of two moves in *Dia. 4*.

*Move value:* In *Dia. 2*, the initial position's *move value* is the difference of the count 14 of the follow-up position in *Dia. 3* created by Black's move and the count 10 of the follow-up position in *Dia. 4* created by White's sente sequence:

$$14 - 10 = 4.$$

*Count:* The initial *count* 10 is inherited from the count 10 of the follow-up position in *Dia. 4* created by White's sente sequence.

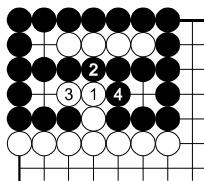
*Verification:* The initial position's local endgame is a local sente as the move value 4 is smaller than the follow-up move value  $4 \frac{1}{2}$  in *Dia. 1* so  $4 < 4 \frac{1}{2}$ .

This ends our naive analysis. Beginners may skip the advanced analysis.

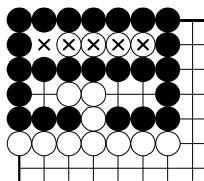


## Advanced Analysis

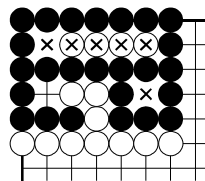
► A long alternating sequence of three or more moves is worth playing successively if the move value is at most each move's gain. To verify or refute an assumption and calculated values of the initial position's local endgame, we must calculate the counts of the positions occurring during the sequences and derive the gains of their moves. With one 'sente sequence', which has an even number of moves, calculate sente values. With two 'gote sequences', which have odd numbers of moves, calculate gote values.



*Dia. 5:* Longest alternating sequence



*Dia. 6:* After move 3:  
Count =  $9 \frac{1}{2}$



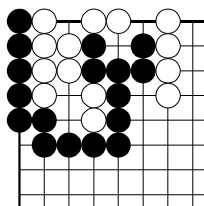
*Dia. 7:* After move 4:  
Count = **10**

To prove the earlier assumption, we must also verify that an alternating sequence of three or four moves is not worth playing successively. For refutations, it is sufficient to calculate the gains of White 3 and Black 4 in *Dia. 5*. The gain of White 3 is the count 10 of the preceding position created in *Dia. 4* minus the count  $9 \frac{1}{2}$  of the succeeding position in *Dia. 6* so  $10 - 9 \frac{1}{2} = \frac{1}{2}$ . The gain of Black 4 is the count 10 of the succeeding position in *Dia. 7* minus the count  $9 \frac{1}{2}$  of the preceding position in *Dia. 6* so  $10 - 9 \frac{1}{2} = \frac{1}{2}$ .

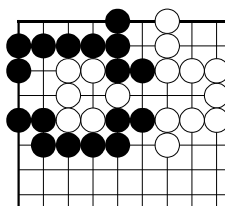
*4-move sequence:* Suppose White's long alternating sente sequence of four moves in *Dia. 5* is worth playing successively, the initial local endgame in *Dia. 2* is a local sente, the count 14 of Black's follow-up position created in *Dia. 3*, the count 10 of White's follow-up position in *Dia. 7* and the sente move value  $14 - 10 = 4$ , which is calculated as the difference. The move value 4 must be at most each move's gain. In particular, the move value must be at most the gain  $\frac{1}{2}$  of Black 4. However, we refute the assumption because the move value is larger than this gain:  $4 > \frac{1}{2}$ .

*3-move sequence:* Suppose White's long alternating gote sequence of the three moves 1 to 3 in *Dia. 5* is worth playing successively, the initial local endgame in *Dia. 2* is a local gote, the count 14 of Black's follow-up position created in *Dia. 3*, the count  $9 \frac{1}{2}$  of White's follow-up position in *Dia. 6* and the gote move value  $(14 - 9 \frac{1}{2}) / 2 = 2 \frac{1}{4}$ , which is calculated as half the difference. The move value  $2 \frac{1}{4}$  must be at most each move's gain. In particular, the move value must be at most the gain  $\frac{1}{2}$  of White 3. However, we refute the assumption because the move value is larger than this gain:  $2 \frac{1}{4} > \frac{1}{2}$ .

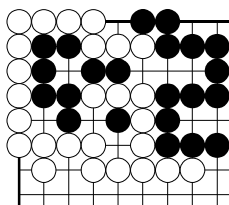
► For most problems with long sequences not worth playing successively, we omit the advanced analyses but provide remarks on gains and long sequences.



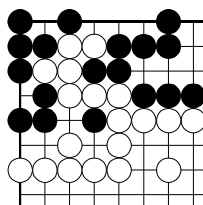
*Problem 57*



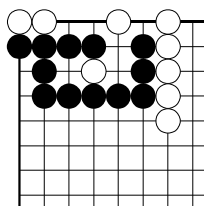
*Problem 58*



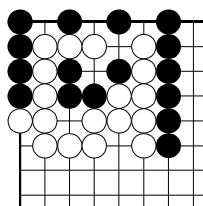
*Problem 59*



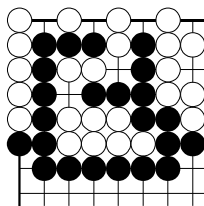
*Problem 60*



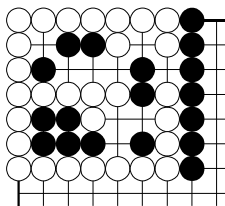
*Problem 61*



*Problem 62*



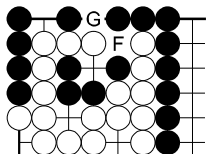
*Problem 63*



*Problem 64*

## Answer 62

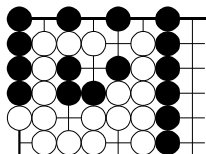
### Follow-ups



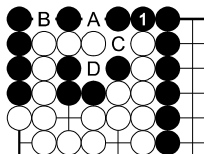
*Dia. 1:* Follow-ups: Move value = 3, count = -15

*Dia. 1:* According to the colour-reversed *Answer 3*, region F is a local sente with the move value 3 and count -7. According to the colour-reversed *Answer 16*, region G is a local sente with the move value 3 and count -8. The *follow-up move value* 3 is the maximum of both individual move values and the local *count* -7 + (-8) = -15 is the sum of both individual counts.

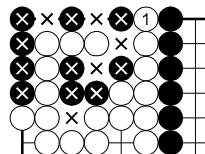
### Initial Position



*Dia. 2:* Initial position: Move value = 5, count = -20



*Dia. 3:* Black starts



*Dia. 4:* White starts: Count = -25

*Dia. 3:* Black 1 creates the follow-up position in *Dia. 1*. - *Dia. 4:* Black has 0 points. We subtract White's 25 points (ten occupied and five empty intersections) so the count is -25.

*Assumption:* We assume that the initial position's local endgame in *Dia. 2* is a local gote and its values depend on Black's follow-up position created by his one move in *Dia. 3*.

*Move value:* In *Dia. 2*, the initial position's *move value* is half the difference of the count -15 of Black's follow-up position in *Dia. 1* and the count -25 of White's follow-up position created in *Dia. 4*:

$$(-15 - (-25)) / 2 = 5.$$

*Count:* The initial local *count* is the average of the count -15 of Black's follow-up position and the count -25 of White's follow-up position:

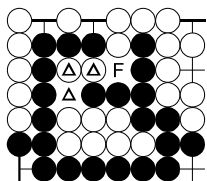
$$(-15 + (-25)) / 2 = -20$$

*Verification:* The initial position's local endgame is a local gote because the move value 5 is larger than the follow-up move value 3, that is,  $5 > 3$ .

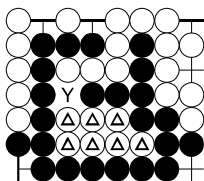
*Remarks:* In *Dia. 3*, Black 1 gains 5, then Black A gains 4, then either Black or White B gains 4, White A gains 3, Black C gains 3 1/2, then either Black or White D gains 3 1/2, White C gains 3. White 1 in *Dia. 4* gains 5. Usually, longer sequences are not worth playing successively.

## Answer 63

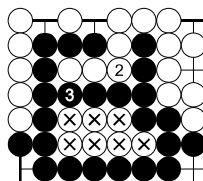
### Follow-up Region F



*Dia. 1:* Region F: Move value = 5, count = 14



*Dia. 2:* Next move value = 7



*Dia. 3:* White's sente continuation: Partial count = 14

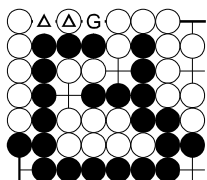
*Assumption:* We assume that region F is a local sente.

*Move values:* In *Dia. 1*, we apply shortcut IV to region F. The two marked occupied intersections contribute  $2 * 2 = 4$  and the one marked empty intersection contributes 1 so the *move value* is 5. We apply shortcut I to region Y in *Dia. 2*, which White 2 in *Dia. 3* creates. The seven marked occupied intersections contribute 7 so the *next move value* is 7.

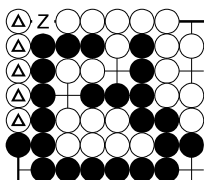
*Count:* The *count* 14 of region F is inherited from the partial count 14 in the follow-up position created by White's sente continuation in *Dia. 3*.

*Verification:* Region F is a local sente because its move value 5 is smaller than its next move value 7, that is,  $5 < 7$ .

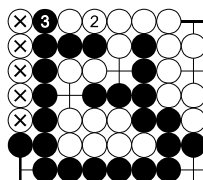
### Follow-up Region G



*Dia. 4:* Region G: Move value = 3, count = 10



*Dia. 5:* Next move value = 5



*Dia. 6:* White's sente continuation: Partial count = 10

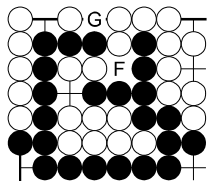
*Assumption:* We assume that region G is a local sente.

*Move values:* In *Dia. 4*, we apply shortcut IV to region G. The one marked occupied intersection contributes 2 and the one marked empty intersection contributes 1 so the *move value* is 3. We apply shortcut I to region Z in *Dia. 5*, which White 2 in *Dia. 6* creates. The five marked occupied intersections contribute 5 so the *next move value* is 5.

*Count:* The *count* 10 of region G is inherited from the partial count 10 in the follow-up position created by White's sente continuation in *Dia. 6*.

*Verification:* Region G is a local sente because its move value 3 is smaller than its next move value 5, that is,  $3 < 5$ .

## Follow-ups Conclusion

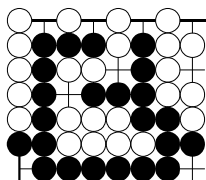


*Dia. 7:* Follow-ups: Move value = 5, count = 24

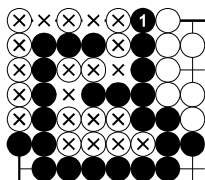
*Dia. 7:* Region F has the move value 5 and count 14. Region G has the move value 3 and count 10. The *follow-up move value* 5 is the maximum of both individual move values and the follow-up position's *count*  $14 + 10 = 24$  is the sum of both individual counts.

► Alternatively, the sente sequences in *Dias.* 3 and 6 can be combined for determining the count 24.

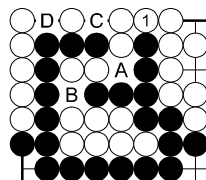
## Initial Position



*Dia. 8:* Initial position:  
Move value = 7, count = 31



*Dia. 9:* Black starts:  
Count = 38



*Dia. 10:* White starts

*Dia. 9:* Black has 38 points (seventeen occupied and four empty intersections) so the count is 38. -  
*Dia. 10:* White 1 creates the follow-ups.

*Assumption:* We assume that the initial position's local endgame in *Dia. 8* is a local gote and its values depend on White's follow-up position in *Dia. 7* created by his one move in *Dia. 10*.

*Move value:* In the initial position in *Dia. 8*, the local gote endgame's *move value* is half the difference of the count 38 of Black's follow-up position created in *Dia. 9* and the count 24 of White's follow-up position in *Dia. 7*:

$$(38 - 24) / 2 = 7.$$

*Count:* The initial *count* is the average of the count 38 of Black's follow-up position and the count 24 of White's follow-up position:

$$(38 + 24) / 2 = 31.$$

*Verification:* The initial position's local endgame is a local gote because the move value 7 is larger than the follow-up move value 5, that is,  $7 > 5$ .

*Remarks:* Black 1 in *Dia. 9* gains 7. In *Dia. 10*, White 1 gains 7, Black A gains 5, White A gains 7, then either Black or White B gains 7, Black C gains 3, White C gains 5, then either Black or White D gains 5. Usually, longer sequences are not worth playing successively.