Modelling Animal Motion as

Random Walkers and Active Brownian Particles

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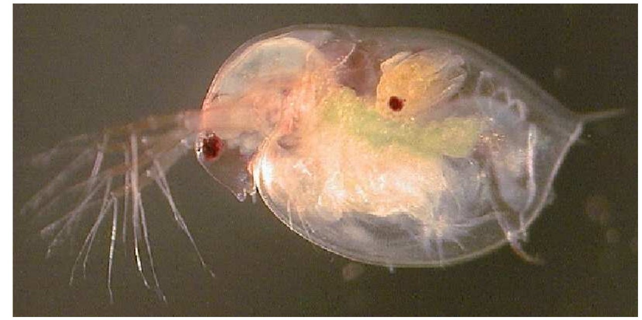
Overview

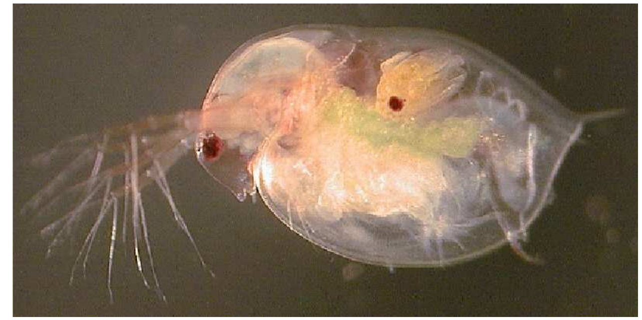


daphnia

- the **animal** and its **movement** features
- What are the advantages to move this way?
- proposing two motion models, a
  - Random Walker
  - Active Brownian Particle

  (Two well known models, not always easily **mapped** onto each other!)
- model of **food** consumption

The Daphnia



daphnia

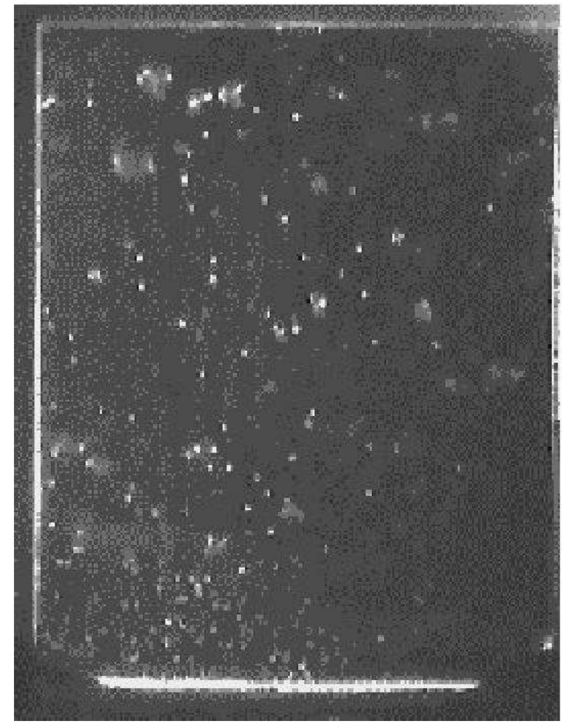
- $2..4\ mm$ long **under water** animal, part of “zooplankton”
- mainly **night-active**
- orientation: optical, mechanical and chemical

The Daphnia

How came *daphnia* into the focus of physicists?

- **swarming** without leading animal under certain light conditions
- **common** direction of rotation around one landmark

for **swarming** see EBELING, ERDMANN, SCHIMANSKY-GEIER, or HERNÁNDEZ-GARCÍA and LÓPEZ...



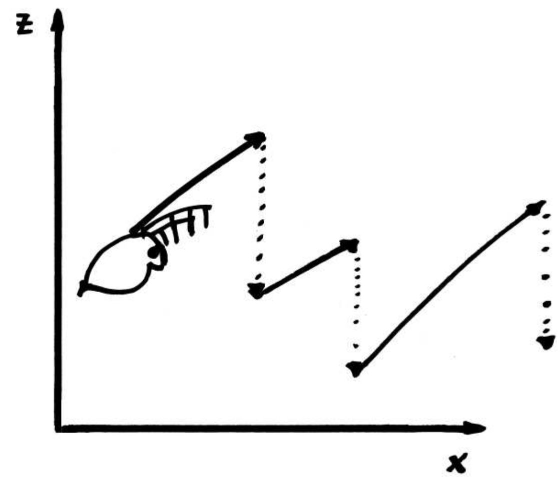
swarming *copepods* by R. STRICKLER with A. OKUBO & J. YEN

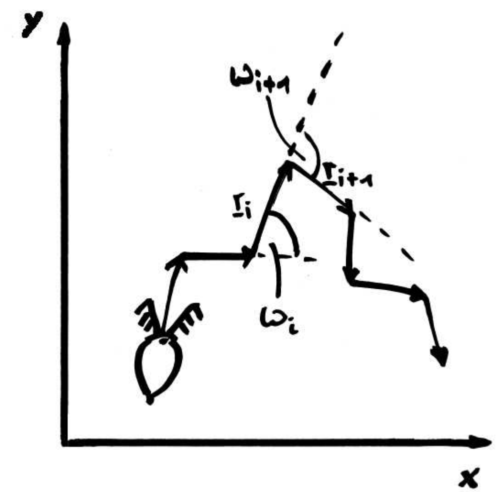
The Observation

here movement in **darkness**, **no swarming**

- 3 swim strokes per second, $v = 4..16\,\frac{mm}{s}$
- in “equilibrium” motion nearly in plane

- measurement of **turning angle distribution** $p(\omega)$ in darkness [a]





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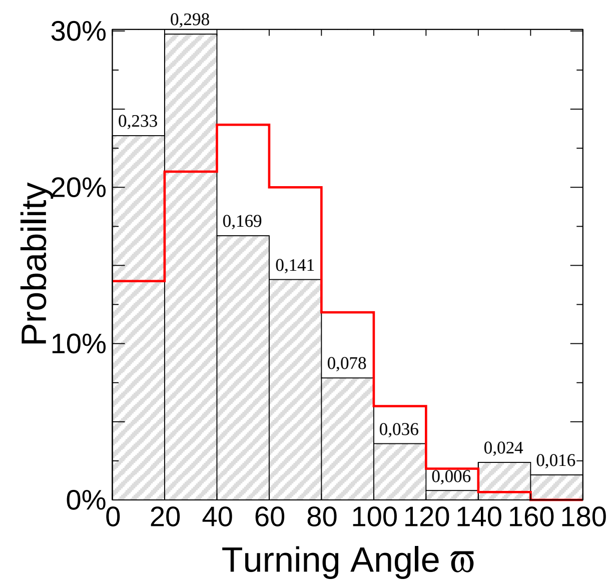
[a] experiment by Frank Moss and Anke Ordemann, St. Louis

The Observation

numbers:

- **preference** of angles between 20° and 40° (30%)
- one 10th goes **backwards**
- Gaussian: $\langle|\omega|\rangle = 48^\circ$, $\sigma = 36^\circ$ (red line)



Proposing a Random Walker Model [a]

- which moves in a **plane** (2 dimensions)
- changes its direction **relative** to the view
- and according to a given **turning angle probability.**

RW are efficiently programmed on a computer.

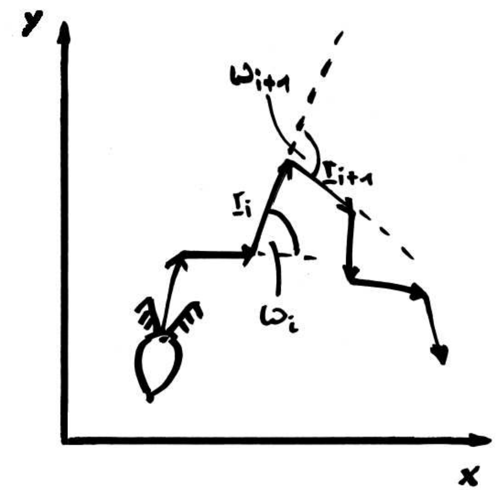
[a] PEARSON, RAYLEIGH; *Nature 1905*

Daphnia as Random Walker

steps of **fixed** length at **discrete** times
here space is **continuous**

- assuming constant step size $\lambda = \langle v \rangle \langle \tau \rangle \approx \frac{10}{3} mm$

$$\vec{x}_{i+1} = \vec{x}_i + \lambda \begin{pmatrix} \cos\left(\phi_{i-1} + \omega_i\right) \\ \sin\left(\phi_{i-1} + \omega_i\right) \end{pmatrix}$$

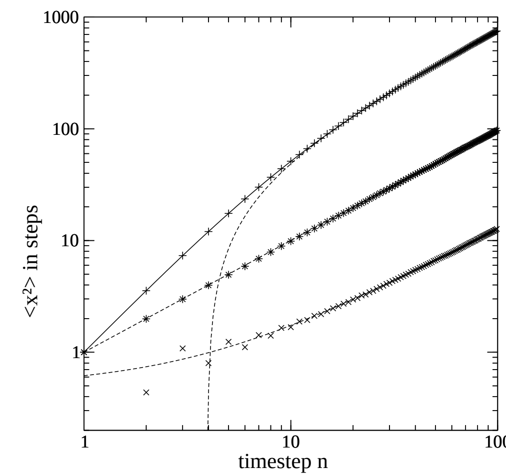


- ϕ: direction of motion
- ω: angle between successive steps, $p(\omega)$-distributed random variable

A Random Walkers $\left\langle x^2\,(n)\right\rangle$

- mean square displacement of this **correlated** Random Walker:

$$\boxed{\Rightarrow \left\langle \vec{x}_n^2 \right\rangle = \lambda^2 \left[n \frac{1+\gamma}{1-\gamma} - 2\gamma \frac{1-\gamma^n}{(1-\gamma)^2} \right]}$$ [a]



with $\gamma := \langle \cos(\phi_i - \phi_{i+1}) \rangle = \int_{-\pi}^{+\pi} p(\omega) \cos \omega \, d\omega$

- for long times (n): **linear** growth of displacement
- **diffusion coefficient** (in 2-d):

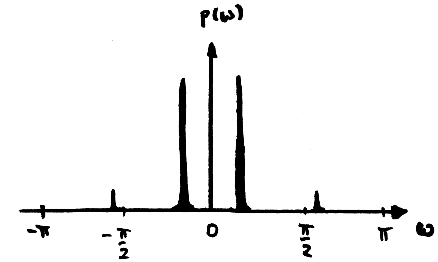
$$4D = \frac{\lambda^2}{\tau} \frac{1+\gamma}{1-\gamma} =: \frac{\lambda^2}{\tau} D_n$$

[a] e.g. in AKIRA OKUBO: “*Diffusion and Ecological Problems: Modern Perspectives*”

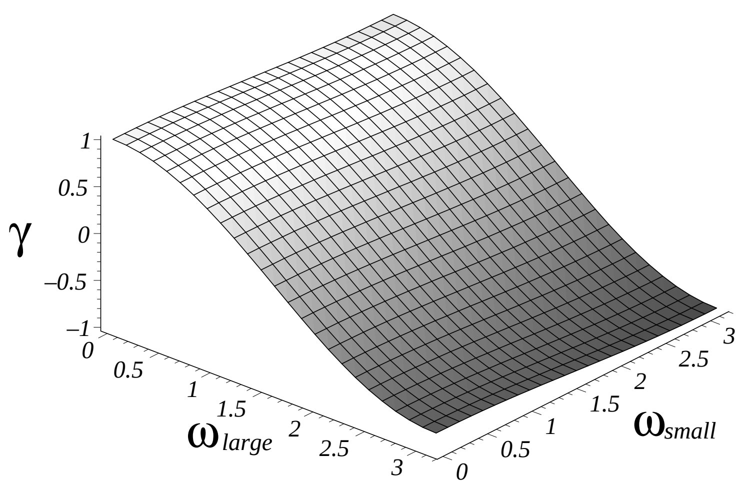
RW: Diffusion Coefficient (a)

Solution for a 2-δ-peak distribution

(to simulate backward jumps)



- $p(\omega) = \frac{1}{2}\left[a\,\delta(|\omega| - \omega_1) + (1-a)\,\delta(|\omega| - \omega_2)\right]$,
  γ easily obtained $\Rightarrow D$
- Position of small δ-peak ($P = \frac{1}{10}$) reduces D by 30% ($\omega_1 = 48^\circ, \omega_2 = 48^\circ..150^\circ$)



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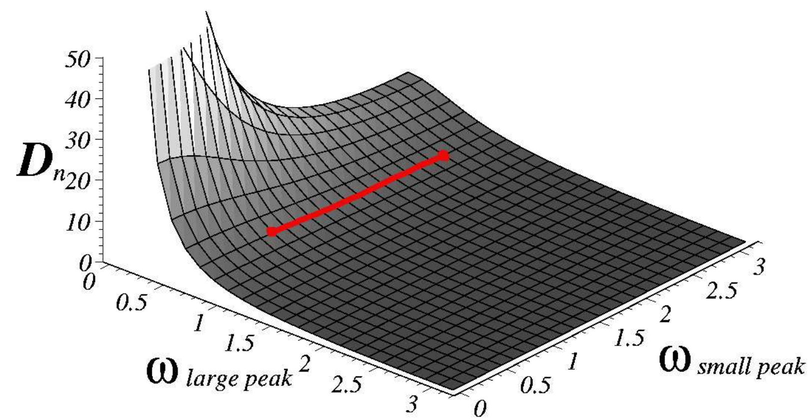
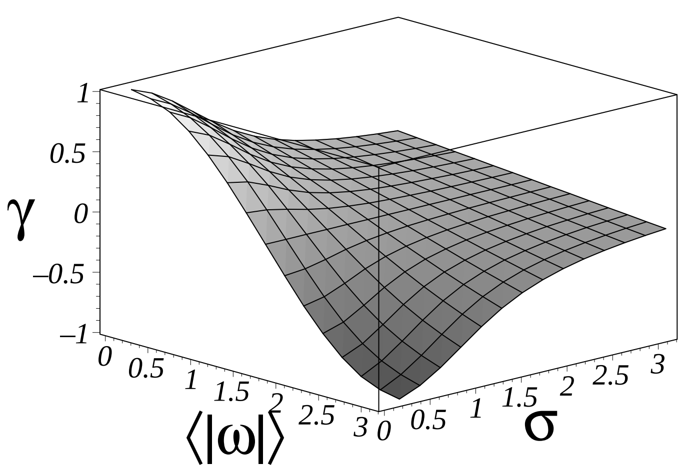


RW: Diffusion Coefficient (b)

Solution for a Gaussian distribution

(to explore preferred angle and its variance)

- solution for Gaussian distributed angles can be derived [a]
  - for $\langle|\omega|\rangle = 48°, \sigma = 36°$ (red line in shown histogram) $D_n = 3.1$
  - i.e. 3 times faster than free particle
  - high gradient area



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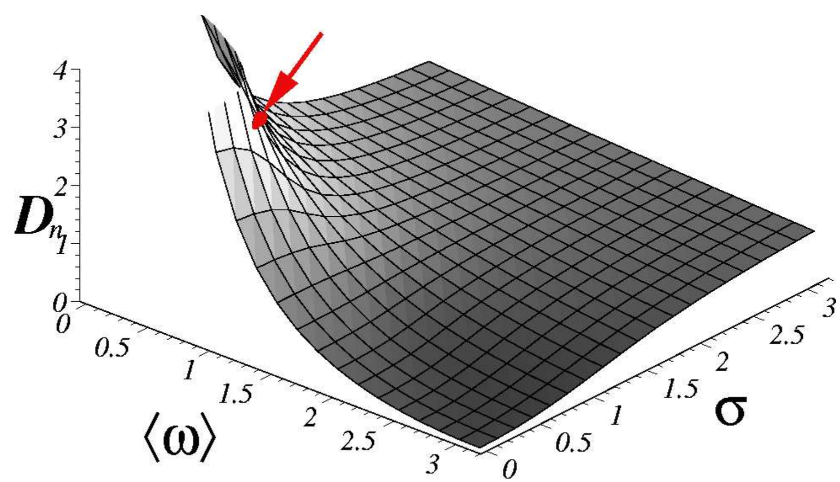

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[a] Komin, Erdmann, Schimansky; *Fluctuation and Noise Letters 03/2004*

Conclusions from Random Walker

- tiny amounts of backward jumps change spreading significantly
- *daphnia* diffusion is 3 times **faster** than unbiased Random Walker
- diffusion coefficient of *daphnia* is situated in **high gradient** area

Proposing an Active Brownian Particle model

- with angular correlation

ABP are active due to **nonlinear friction** (*“self-propelled particles”*).

Cost of implementation higher than that of RW, but **interactions** are more easily introduced.

Daphnia as ABP

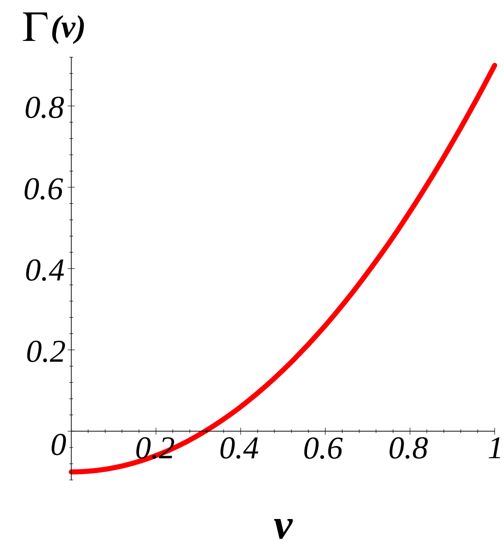
Langevin equation:

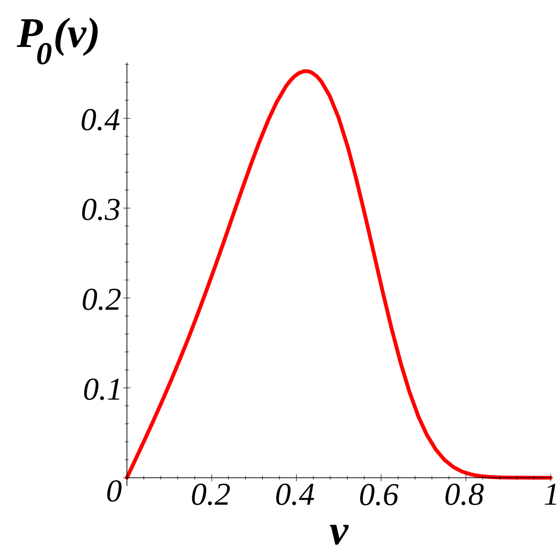
$$\frac{d\vec{r}}{dt} = \vec{v}\,; \qquad \frac{d\vec{v}}{dt} = (\alpha - \vec{v}^2)\vec{v} + \sqrt{2D_v}\vec{\xi}(t) \;\; + \vec{\omega} \times \vec{v}$$

- non-linear friction $\Gamma\,(v)$, accelerates slow particles, slows down fast
- very low noise: $|v| \to \sqrt{\alpha}$
- stationary v**-distribution**:

$$P_0\,(v) \;=\; N\,v\,\exp\left(-\frac{1}{D_v}\int \gamma(v)\,v\,dv\right)$$

- $\vec{\omega} = \{0, 0, \omega\}$ induces Lamor-like rotation, no influence on v**-distribution**





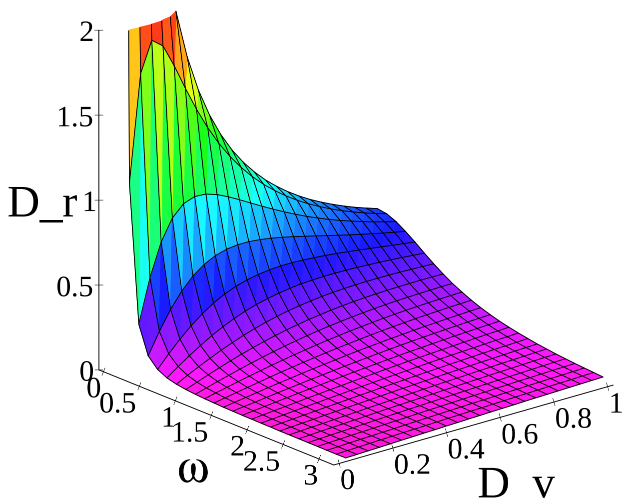
Diffusion Coefficient of *ABP*

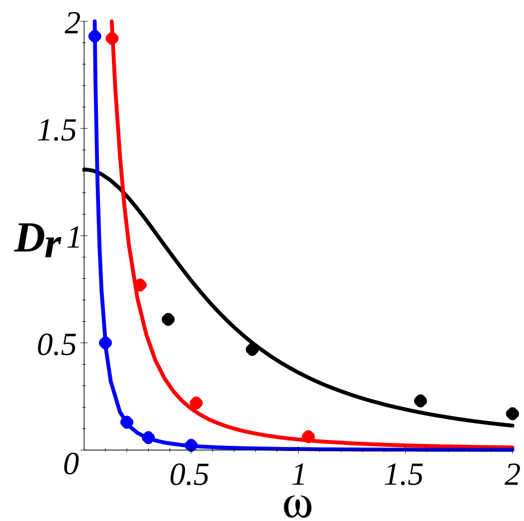
Approximation:

- $D_v/\alpha \to 0$, all particles move with most probable $\tilde{v}$

$$\Rightarrow \boxed{D_r \;=\; \frac{\tilde{v}^4}{2D_v}\left(\frac{1}{1+\omega^2\,\tilde{v}^4/D_v^2}\right)}\,^{a}$$

Simulations (dots): $\alpha = 1$; $D_v = 1$ (black), $D_v = 0.1$ (red), $D_v = 0.01$ (blue)





[a] Schimansky, Erdmann, Komin; *Physica A 10/2004*

Conclusions from ABP

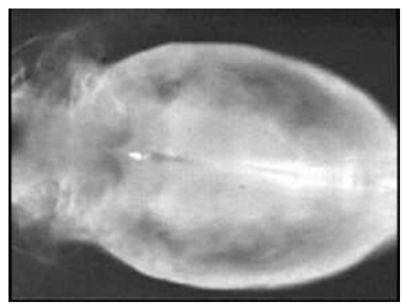
- diffusion coefficient for proposed model obtained
- for low noise good accordance to simulation data
- D dependence qualitativly similiar to RW

BUT:

- not a one-to-one “translation”

Proposing a food model

- We know that turning angle distribution has strong influence on D.
- How do differently diffusing particles conquer their territory?



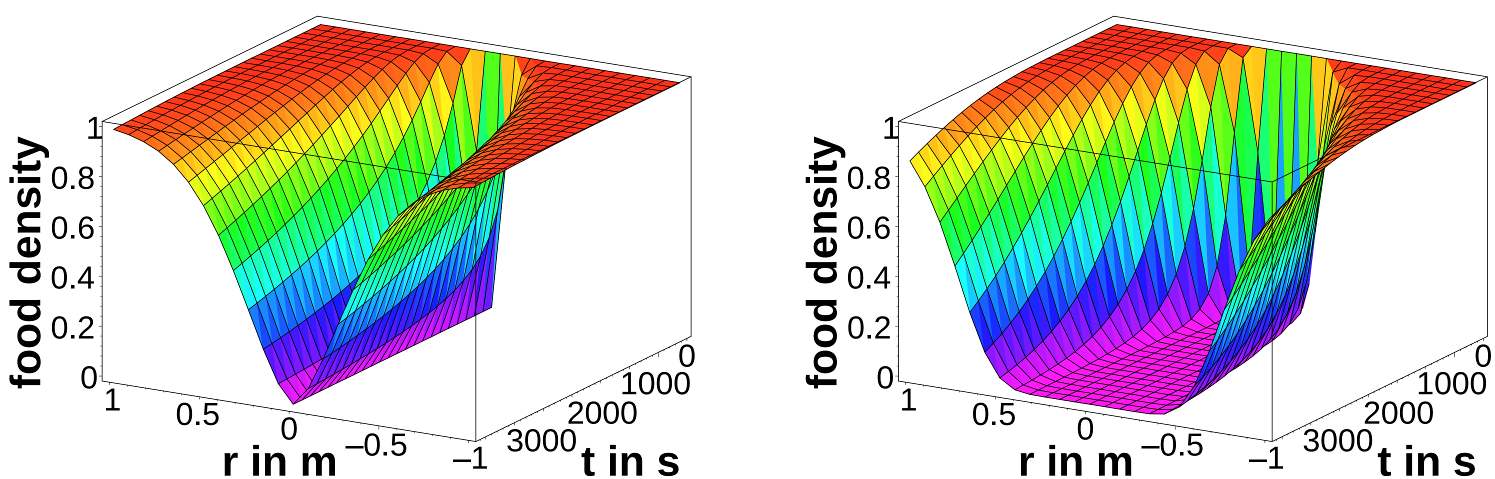
Food Model

- food density $f(\vec{r},t)$, food uptake rate ρ, *daphnia* density $C(\vec{r},t)$

$$\partial_t f(\vec{r},t) = -\rho\, f(\vec{r},t)\, C(\vec{r},t)$$

Assumptions:

- food does neither move nor grow, *daphnia* density is a Gaussian
- integration yields: $f(\vec{r},t) = f_0 \exp\left[-\frac{\rho}{4\pi\, D}\mathrm{E}_1\left(\frac{\vec{r}^2}{4Dt}\right)\right]$



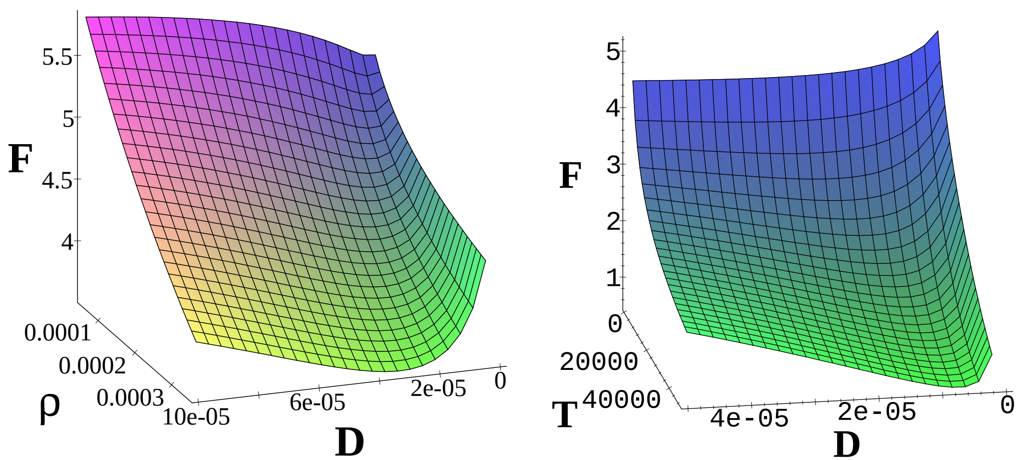
left: $\rho = 2.5 \cdot 10^{-4}\,\frac{m^2}{s}$, right: $\rho = 2.5 \cdot 10^{-3}\,\frac{m^2}{s}$

Food Consumption

- leftover food in a circle (of radius R) for the time T:

$$\boxed{F_{R,T,\rho}(D) = 2\pi f_0 \int_0^R \exp\left[-\frac{\rho}{4\pi\, D}\mathrm{E}_1\left(\frac{\vec{r}^2}{4DT}\right)\right] r dr}$$

- there is a **minimum** in terms of D
  - with constant area (R) and time (T)   (left picture)
  - as well as with constant area (R) and uptake rate (ρ)   (right picture)



Conclusions

The *daphnia* movement was modelled in two different ways.

- Random Walker model:
  - backward jumps reduce D by 30%
  - *daphnia* diffuse (3 times) faster than Brownian RW
  - D is in a high gradient area
- Active Brownian Particles: long time scale solution given

An exact mapping RW $\leftrightarrow$ ABP is desired.

The developed food model

- shows local maximum in consumed food regarding D (and ω).

The food model parameters (area R, time T, uptake rate ρ) should be **stated** more precisely (real dimensions) and **measured** and **compared** in biological experiment.