Vortrags-Vorspann

A random-module in action

Kollision-M.pl2

The power of modules

("building stones")

in teaching mathematics

Dresden, ACDCA 2006

My logo

Used Software:
- Voyage 200 -
- Animato -
- Derive -

Dr. Eberhard Lehmann, Berlin

mirza@snafu.de --- www.snafu.de/~mirza

The starting point of the reflections is the module-triangle

The module-triangle

and it´s application in teaching

Define modules
by your own -
by the teacher
by the students

Use modules
(modules of your own
or of the CAS-System)
Use modules of modules

Analyse modules
(with experiments)

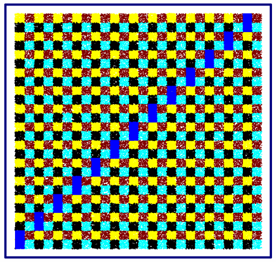
Some special examples →

A work of art – using a random-module

Kreisbüschel2-Kunst.pl2

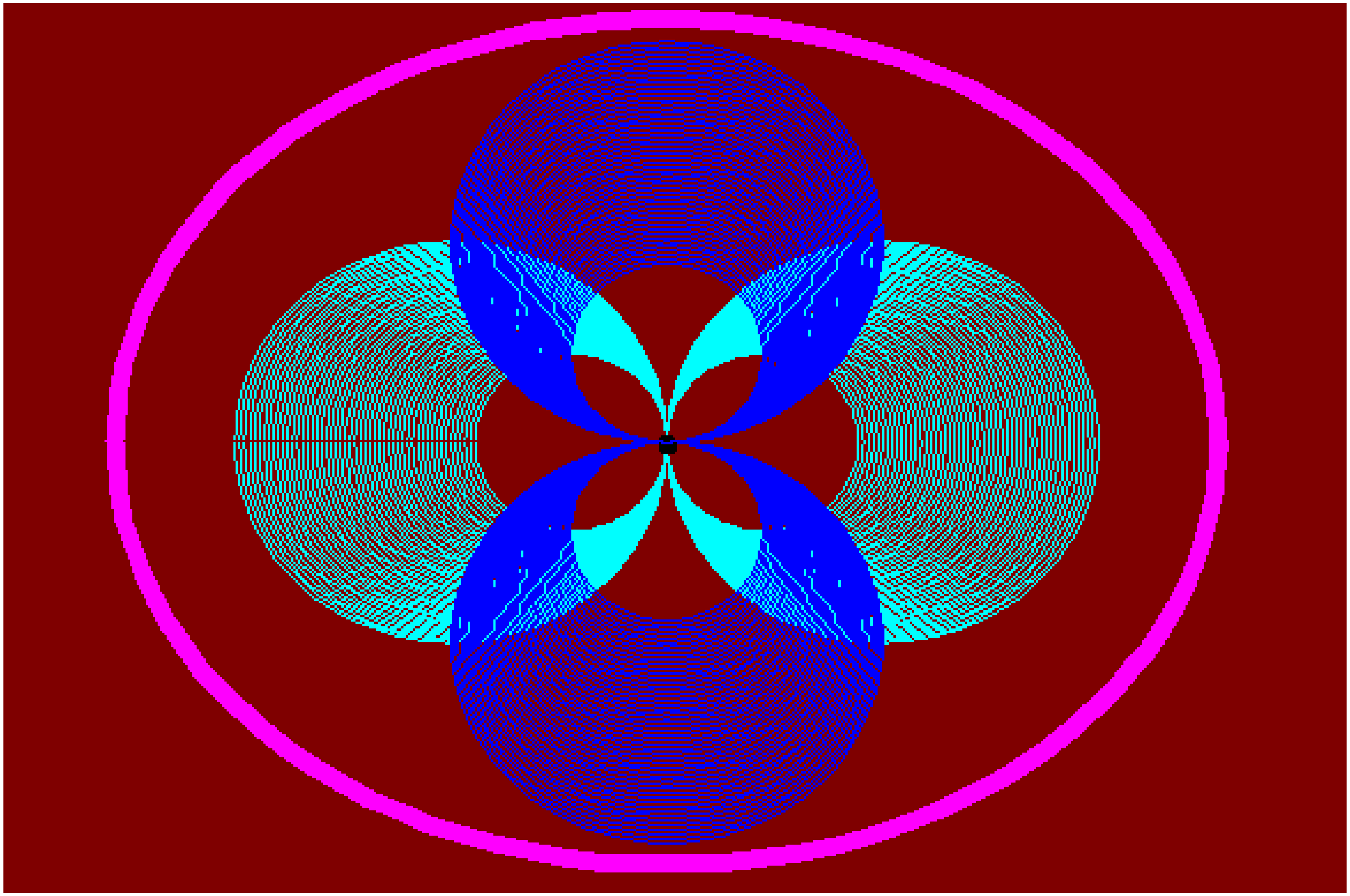
Flaeche6rot.pl2

Zufallsrecheckfeld.pl2



A random-square-field

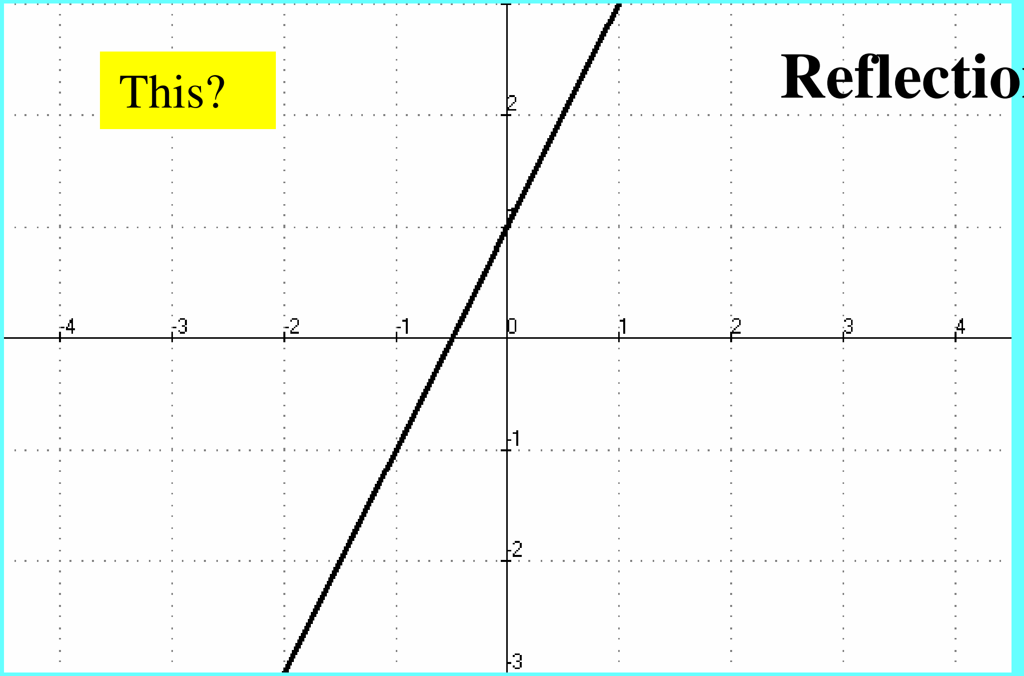
Kreisbüschel2-Kunst.pl2

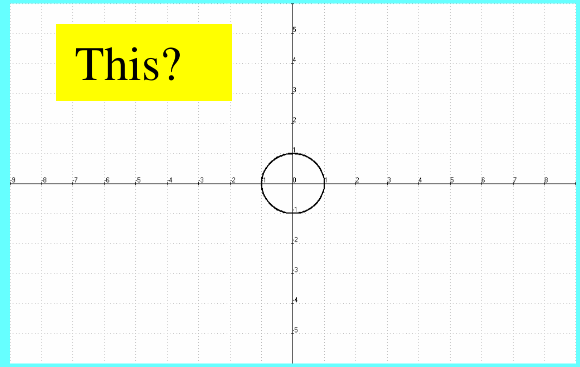


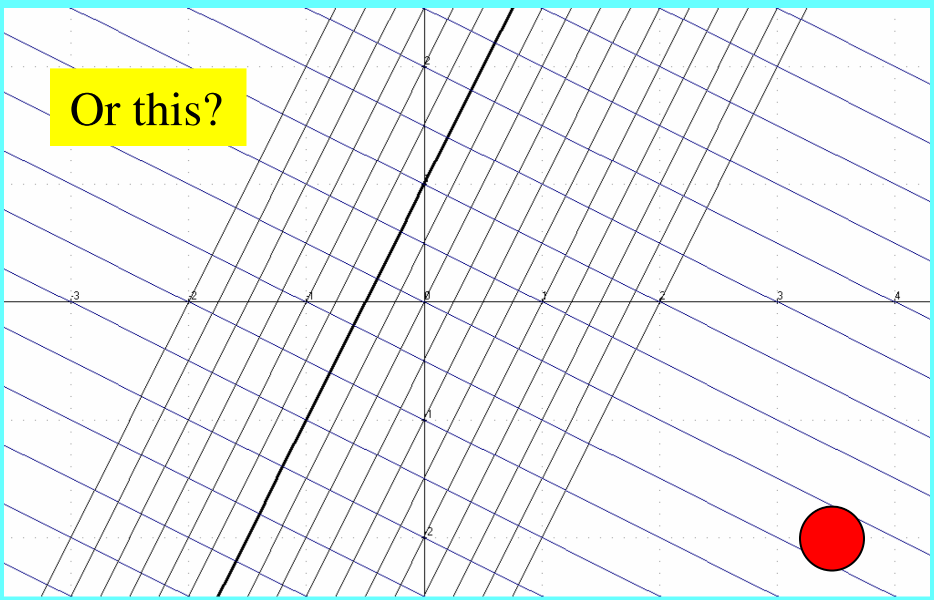
Using a circle-module: Many circles through one point!

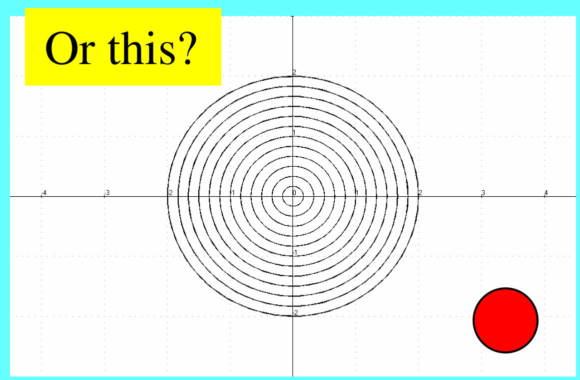
Reflections

Which picture is more provoking for questions of the students?









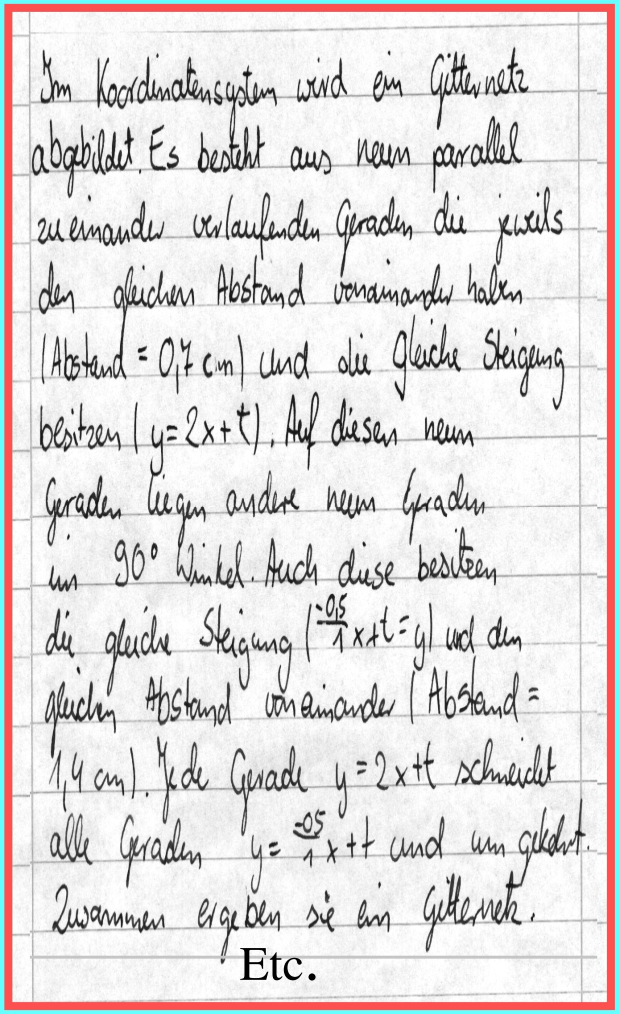
Why?

Because there are many objects - the students can

- compare the objects
- find relations between the objects
- formulate questions about the objects

Such pictures are opened questions, they

- can initiate longer projects or essays about them
- are suitable for new forms of teaching
- **initiate the use of parameters and modules**



Im Koordinatensystem wird ein Gitternetz abgebildet. Es besteht aus neun parallel zueinander verlaufenden Geraden die jeweils den gleichen Abstand voneinander haben (Abstand = 0,7 cm) und die gleiche Steigung besitzen ($y=2x+t$). Auf diesen neun Geraden liegen andere neun Geraden im 90° Winkel. Auch diese besitzen die gleiche Steigung ($\frac{-0,5}{1}x+t=y$) und den gleichen Abstand voneinander (Abstand = 1,4 cm). Jede Gerade $y=2x+t$ schneidet alle Geraden $y=\frac{-0,5}{1}x+t$ und umgekehrt. Zusammen ergeben sie ein Gitternetz.

Etc.

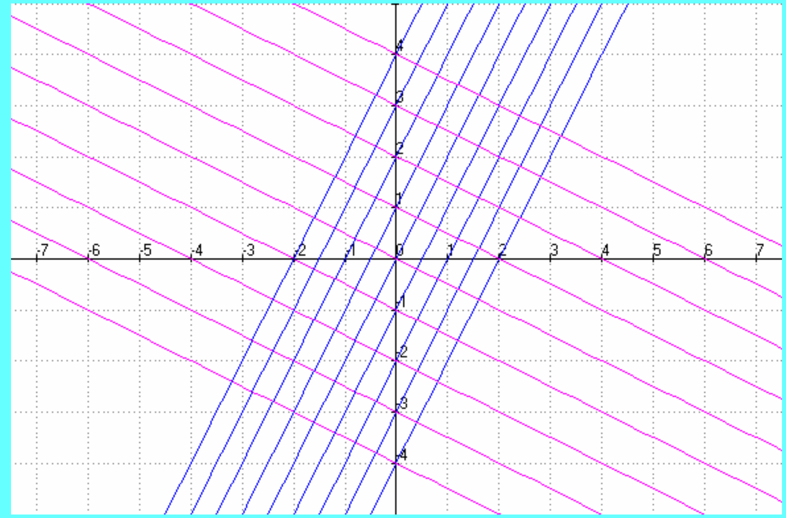
An essay (one hour)

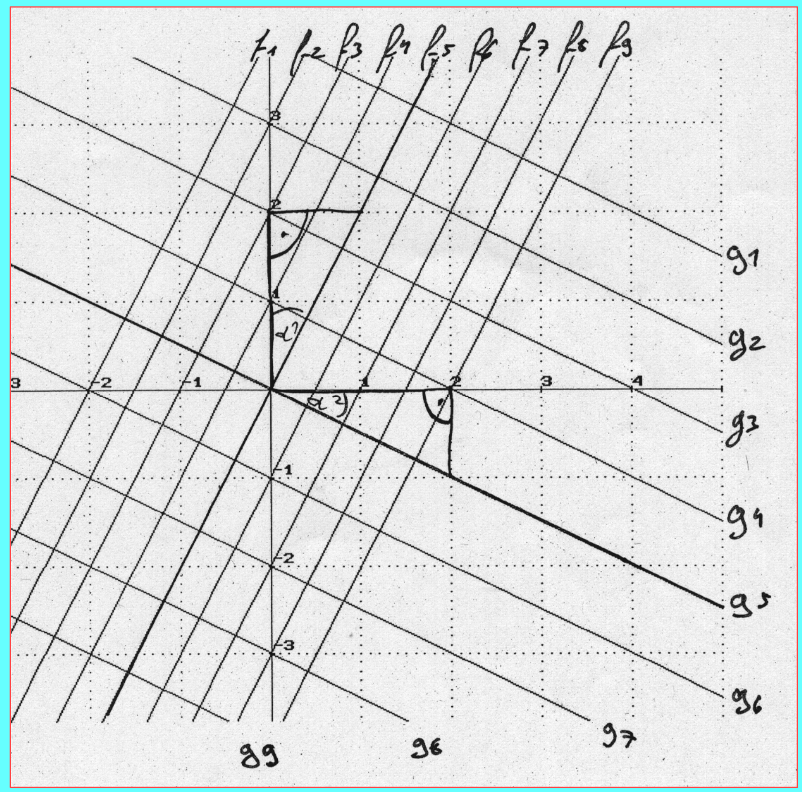
The picture is given. Write a composition on it! (age of the students: 15)

A line-module

f1: a*x+b

f2: f1(2,u)
f3: f1(-1/2,u)





Geraden-parallele-senkrechte.pl2

The modular competence

A quotation by

Helmut Heugl (Austria)

Dr. Helmut Heugl, Anaheim, April 2005

Technology – Standards – Assessment

The influence of the use of Technology in Standards and Assessment

In this lecture he sayed about

Modular competence

Using modules is not new for the learners. Every formula used by the pupils can be seen as a module. While the modules of traditional math education mostly are the starting point for calculations, the CAS-modules often also do the calculations.

W. Dörfler [Dörfler, 1991, pp71] calls any module

"knowledge-unit"

in which knowledge is compressed and

in which operations can be recalled as a whole package.

An important statement: Formulas are modules!

Bino(a,b,n)= $(a+b)^2 = a^2+2ab+b^2$

$(a-b)^3 = a^3+3a^2b+3ab^2+b^3$

Look at the parameters!

f(x,m,n)= $f(x) = m*x+n$

g(x,a,b,c)= $g(x) = a*x^2+b*x+c$

Pytha(a,b,c)= $a^2+b^2=c^2$

V(r,h)= $V = 2\pi r*(r+h)$ Zylindervolumen

Abst(ax,ay,bx,by) $|AB| = \mathrm{sqrt}((ax-bx)^2+(ay-by)^2)$

usw.

The use of modules is´t new for teaching mathematics!

Examples for modules (Voyage 200)

straight line-module define gerade(x,a,b,m) = b+m*(x-a)

(own definition) *Animato: Geraden-durch-3-1.pl2*

Animato: Geraden-durch-a-b.pl2

Plot2.exe

difference-quotient define diffqt(x,h)= ((f(x+h)-f(x)) / h

(own definition) *Animato: DifferQuot-Animation.pl2*

Sum and Sequence sum(seq(i^3-i^2,i,1,5)) 170

(definition by system) expand(seq((a+b)^n,n,1,3)

{a+b, a^2+2ab+b^2,a^3+3a^2b+3ab^2+b^3}

binomic formulas define binomi(a,b,n) = (a+b)^n

(own definition)

What is new? - It is

(1) **the focus** looking at the parameters in the well known formulas!

(2) the use of the **special possibilities of CAS** to work with modules (with parameters) – in Algebra and Geometry.

(3) **a new method** for problem-solving

(4) **a new kind of teaching**

The modul-triangle

- **take system-modules or define modules by your own**
- **solve problems with modules**
- **analyse modules**

Line-module

Zeichne (an einem Computer) möglichst viele Geraden durch den Punkt P(3, 1)!

Lösung: Die Schüler zeichnen in der Regel zunächst einige sich sofort anbietende Geraden, etwa mit den Gleichungen y = 1, y = x–2 (Parallele zu y = x). Es dauert nicht lange bis weitere Geraden eingetragen werden und die Frage nach einer Formel entsteht. Diese wird dann als Baustein definiert:

Punkt (3,1), Steigung m

y–3 = m*(x–1)

y = 3+m*(x–1)

Als Baustein im Voyage-Taschencomputer:
3+m*(x–1) → ger1(x,m,3,1)

Plot2.exe Geraden-Eingangsbild.pl2

General: b+m*(x-a) → ggl(x,m,a,b)

Example for a call: y1(x) = gl(x,m,3,1)

Therefore you can draw a bunch of straight lines at every point in the coordinate-system.

Variationen: Viele Fragestellungen zu Geradenbüscheln

a) Many lines through the point P(3,4).

b) Many lines through the point P(a,b).

c) A bunch of straight lines through durch 18 points on a circle.

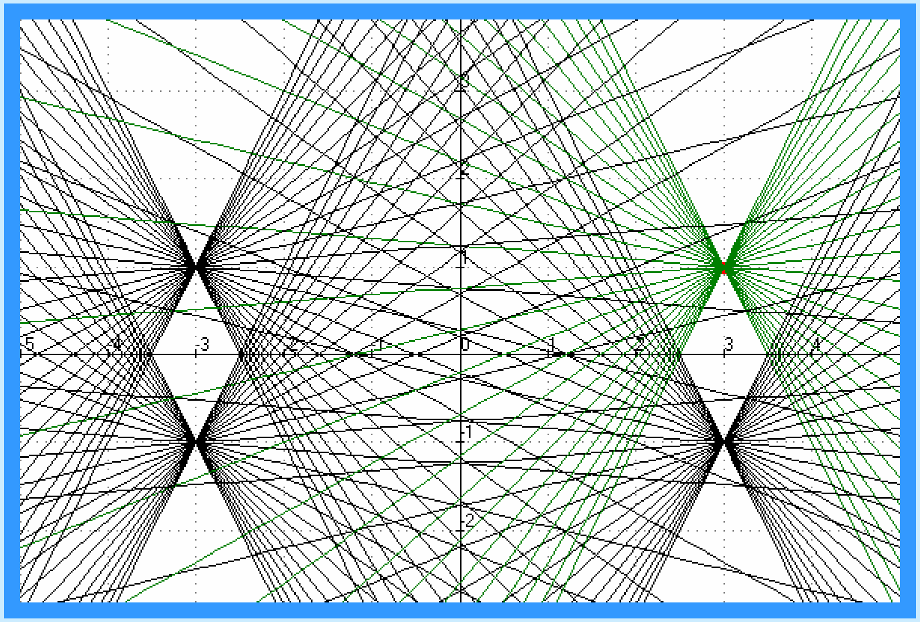
d) Vergrößere dein Bild zu c.

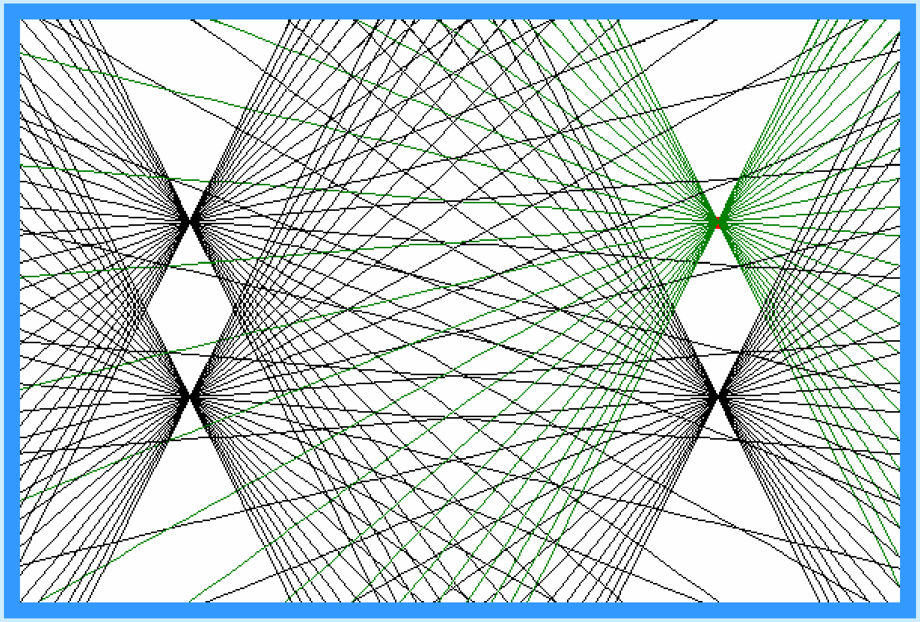
e) Rekonstruiere Bild xxx und wähle andere Positionen der Büschelzentren.

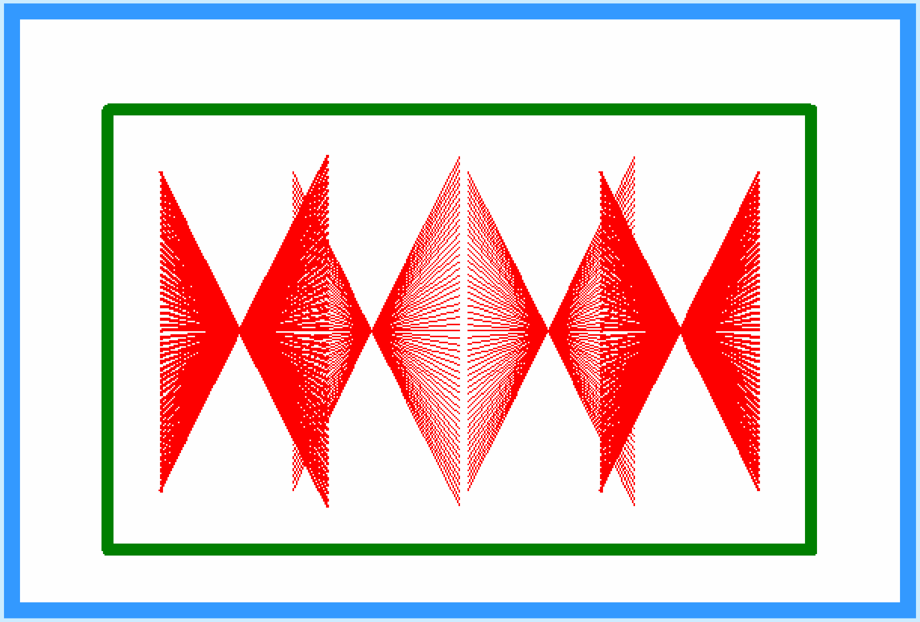
f) Formuliere ähnliche Fragenstellungen für Büschelzentren auf einer Parabel.

g) Formuliere ähnliche Fragenstellungen für Büschelzentren auf selbstgewählten Kurven.

h) Erstelle ein kunstvolles Bild aus Geradenbüscheln.

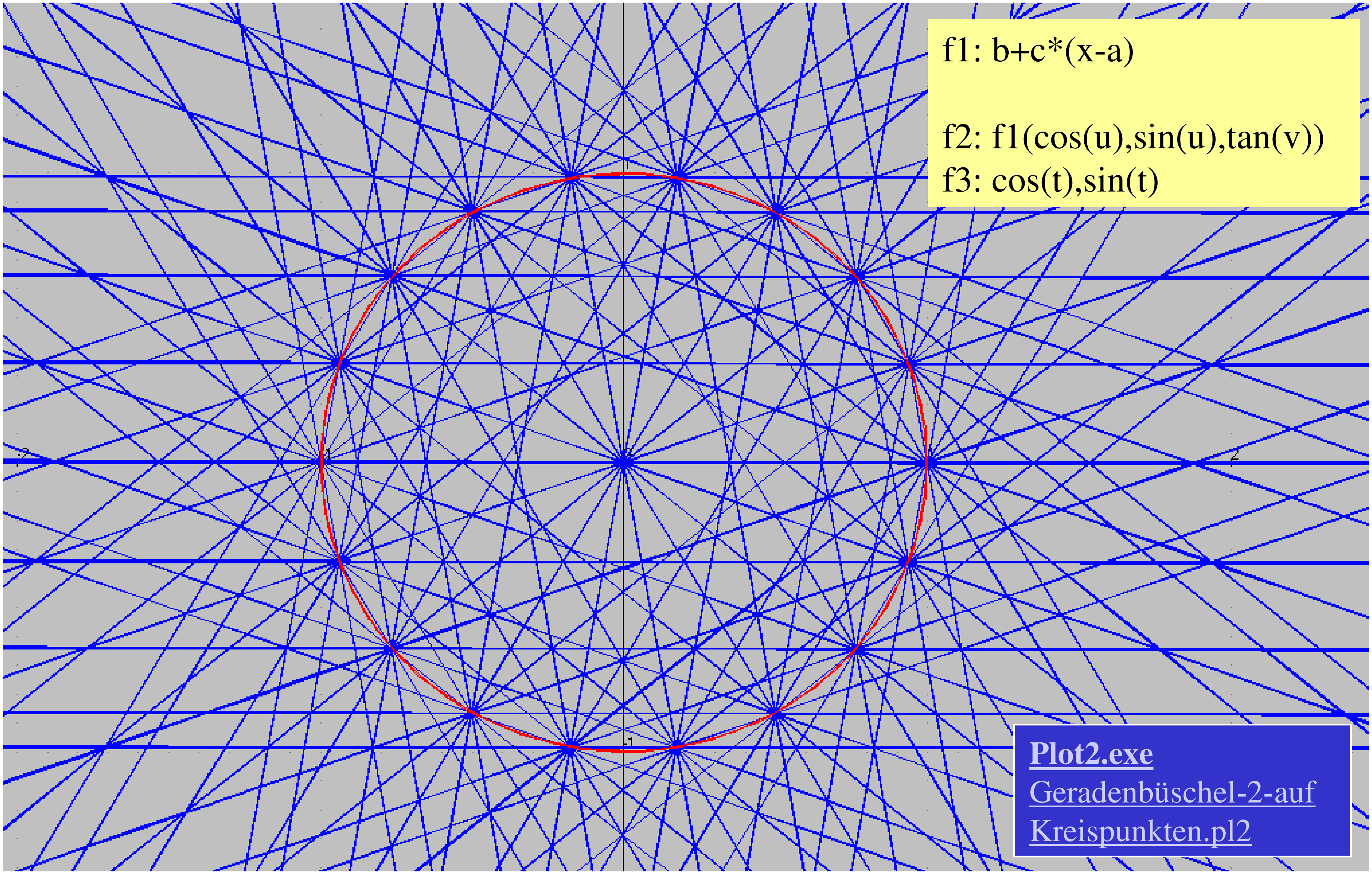




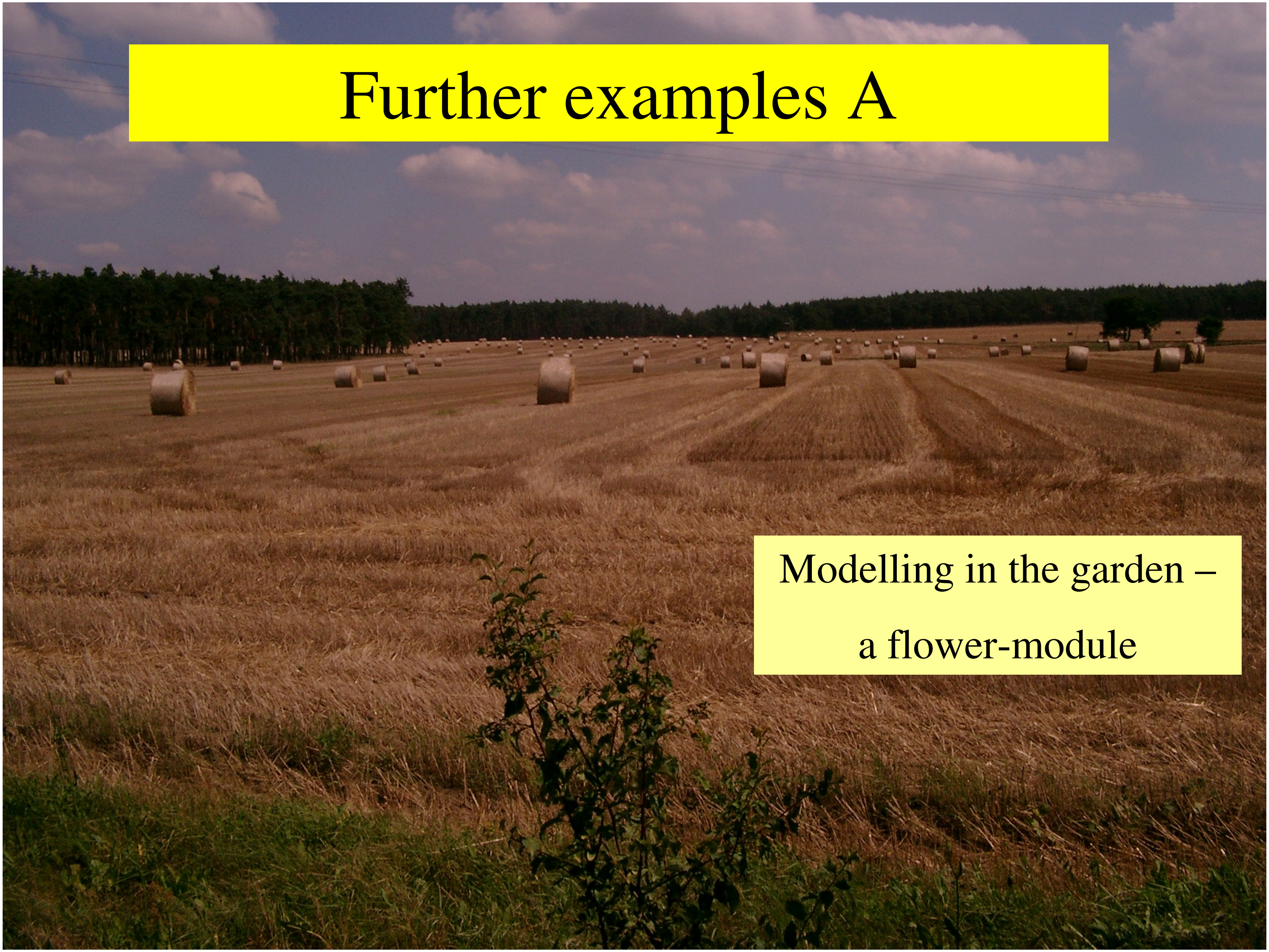


Plot2.exe

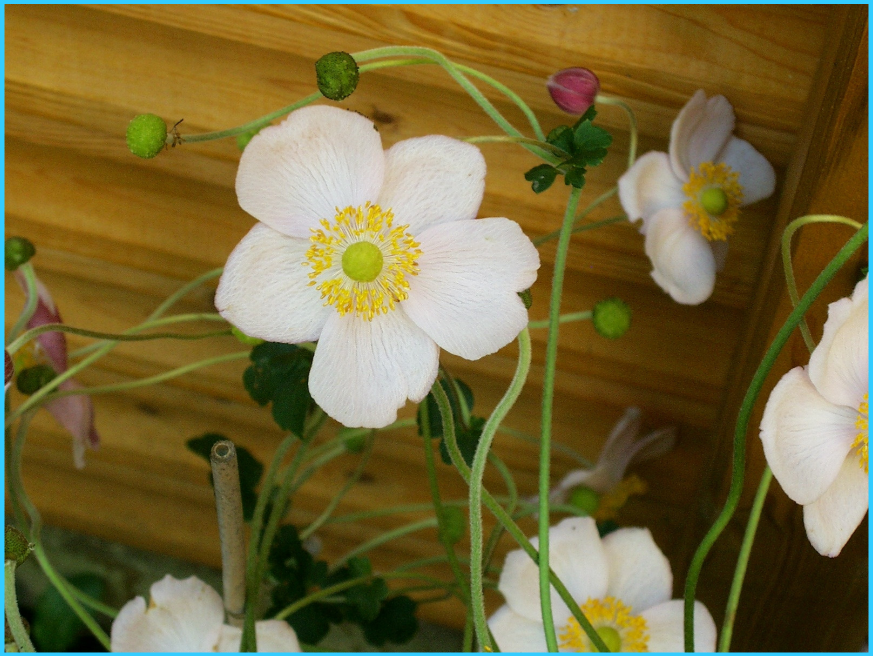
Geraden-durch-a-b-Kunst.pl2



The bunch-centers are lying on a circle

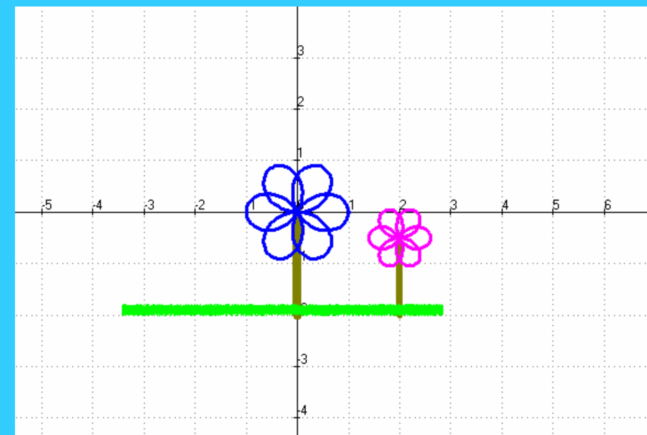

Further examples A
Modelling in the garden –
a flower-module

Mathematics in the garden – with modules



The same structure!

reality | the first modell | the second modell



Film

Our way to a flower-module

The flower-module

f2: a*(cos(b*t)+cos(t))/2+c   **x(t)**

f3: a*(sin(b*t)-sin(t))/2+c   **y(t)**

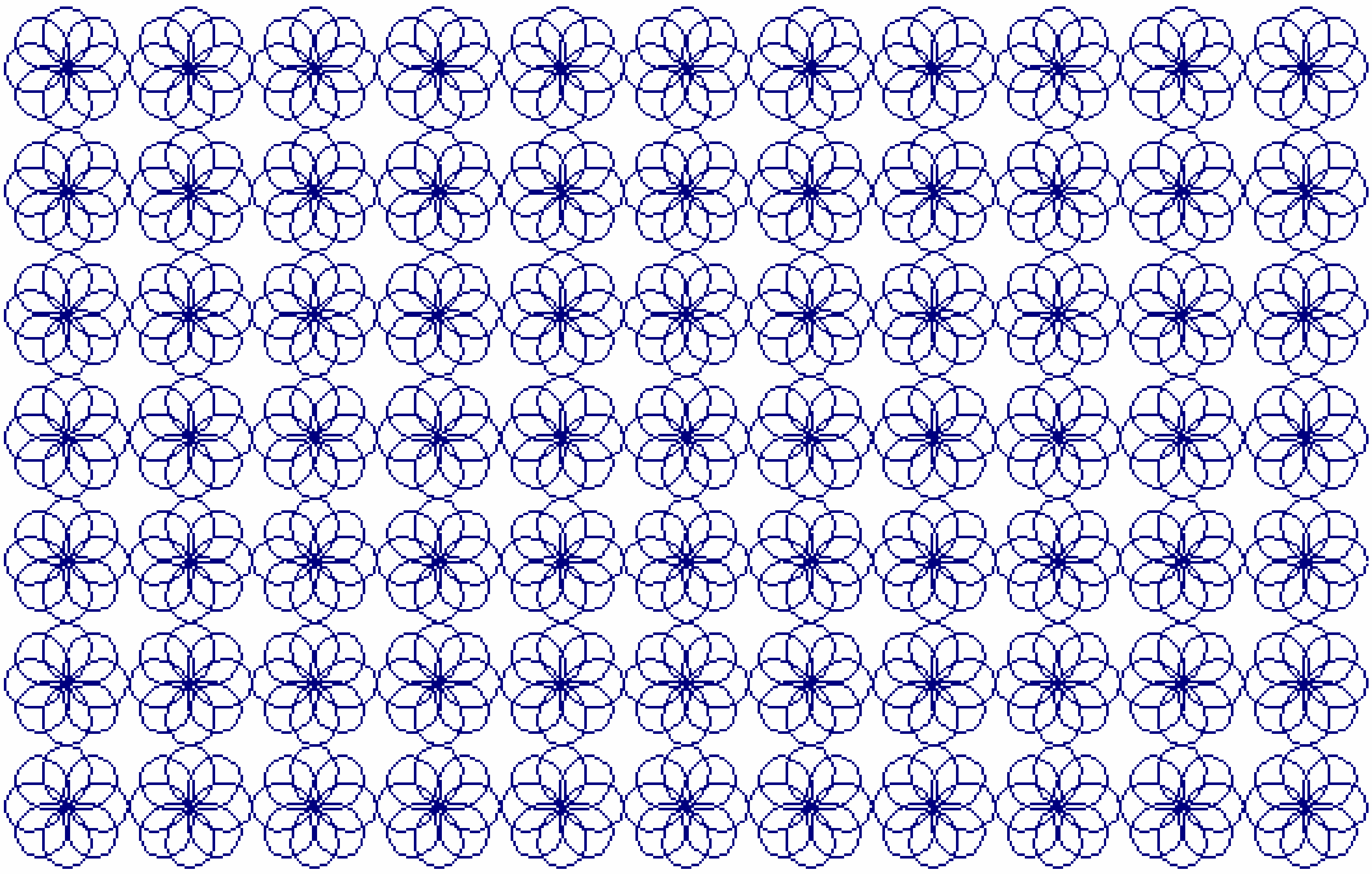
Module-calls

f4: f2(1,5,0),f3(1,5,0)   a special flower

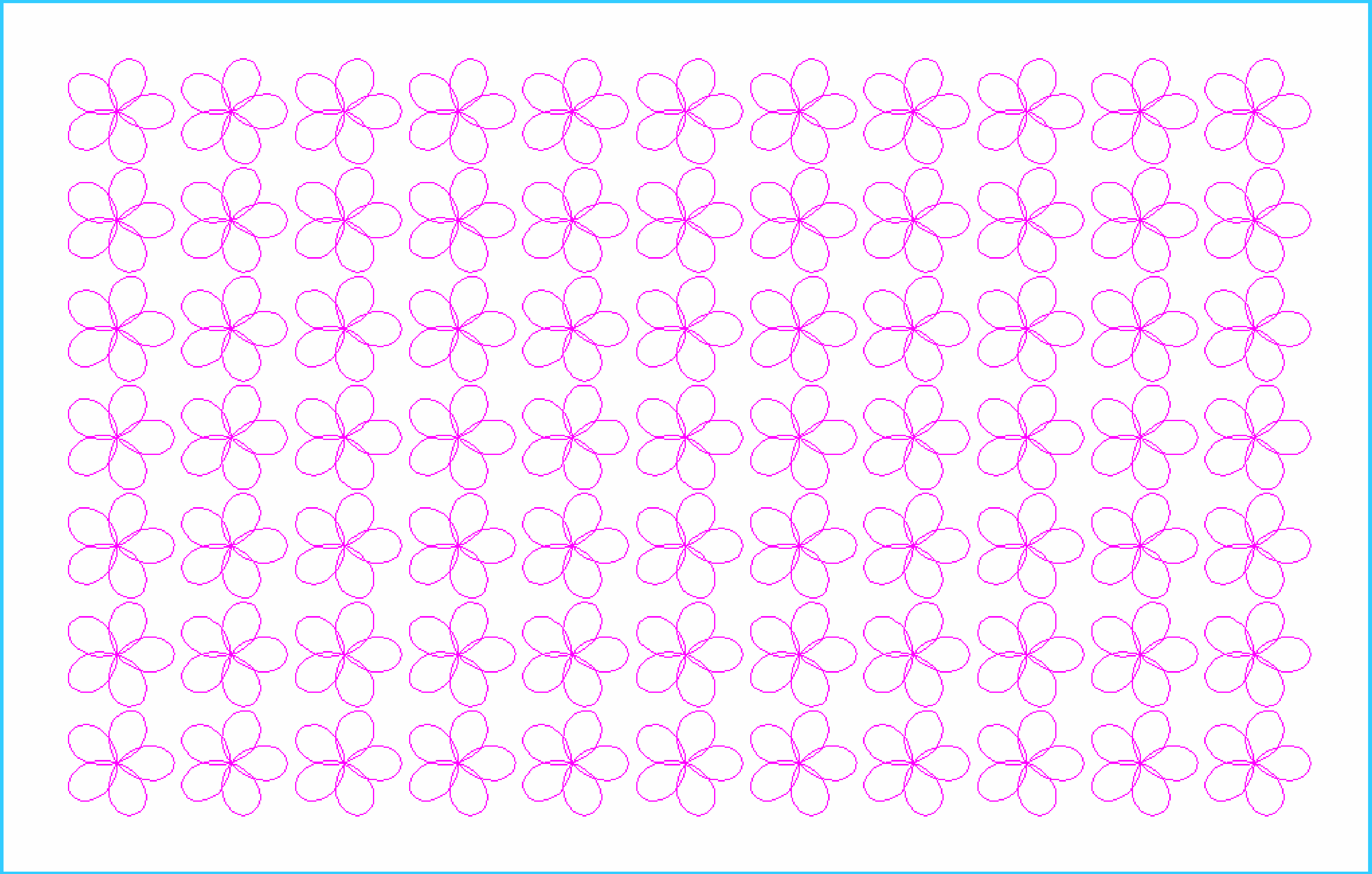
f5: f2(0.6,5,2),f3(0.6,5,-0.5)   another special flower

f6: f2(0.5,4,u),f3(0.5,4,v)   the first flower-field

Application 1 of our module



Application 2 of our module



and so on → a look to my flower-field

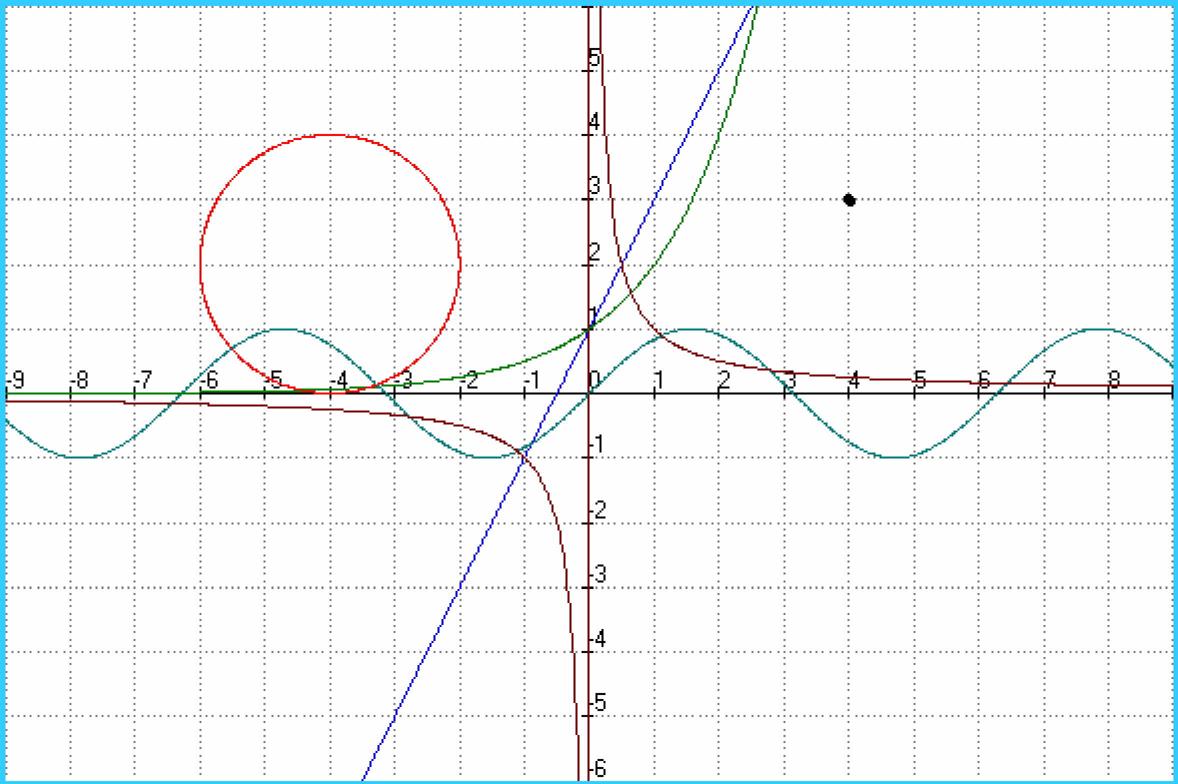
Animato-Datei: Blütenmeer.pl2

Application:

A pattern of flowers on tile in Portugal

A distance-module

What´s the shortest distance of the point P(4,3) to the graphs?



Abstandsberechnung.pl2

These functions are given

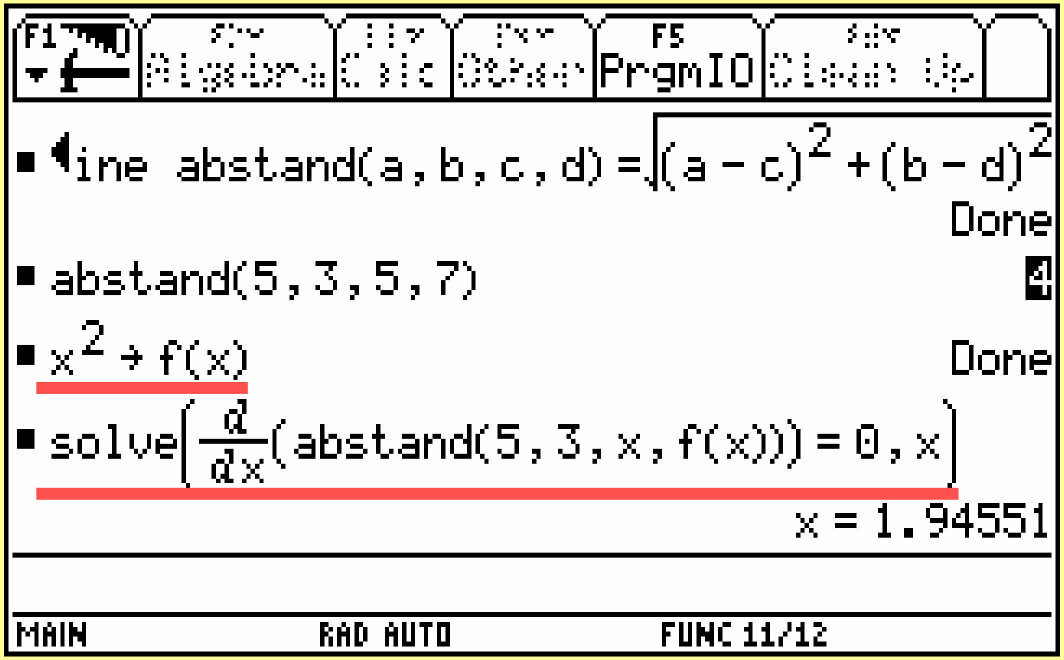
$f1(x) = \sin(x)$,
$f2(x) = 2^x$,
$f3(x) = (x + 1)/(x + 4)$,
f4: $x(t) = 2*\cos(t) + 9$,
$y(t) = 2*\sin(t) + 6$,

The given point is P(4, 3).

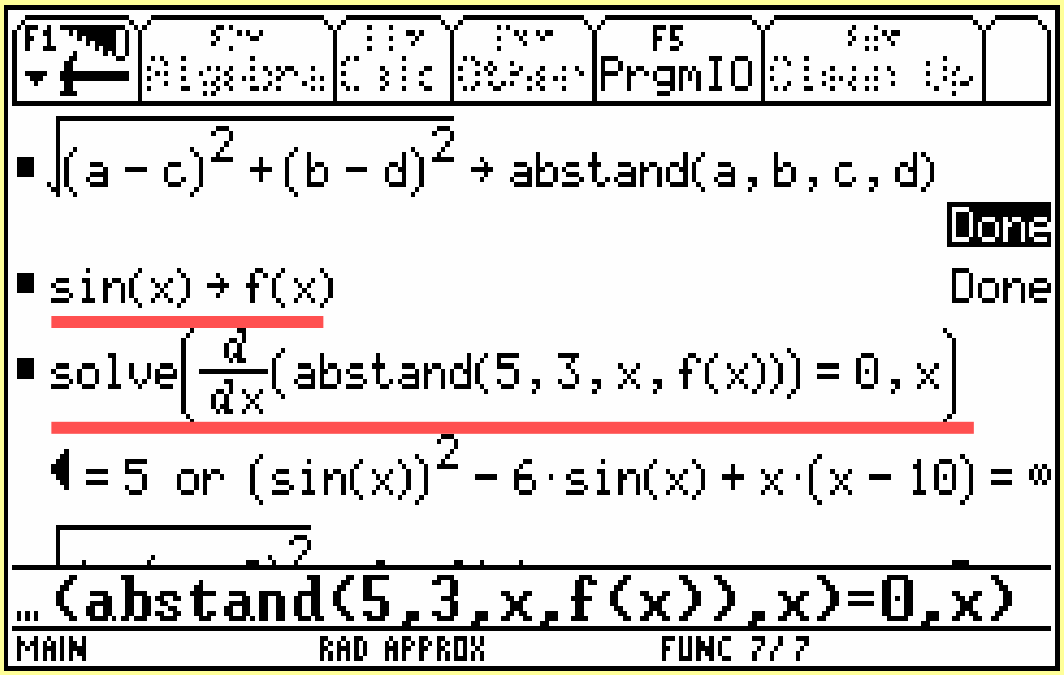
(5 groups with 4 students)

The distance-module

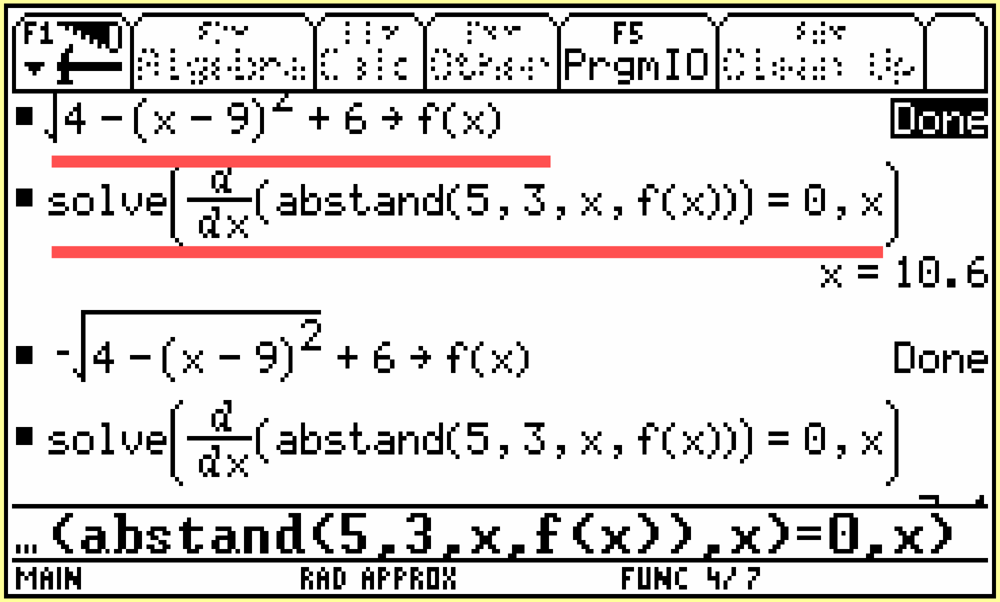
f15: sqrt((a-c)^2+(b-d)^2), Abstandsbaustein

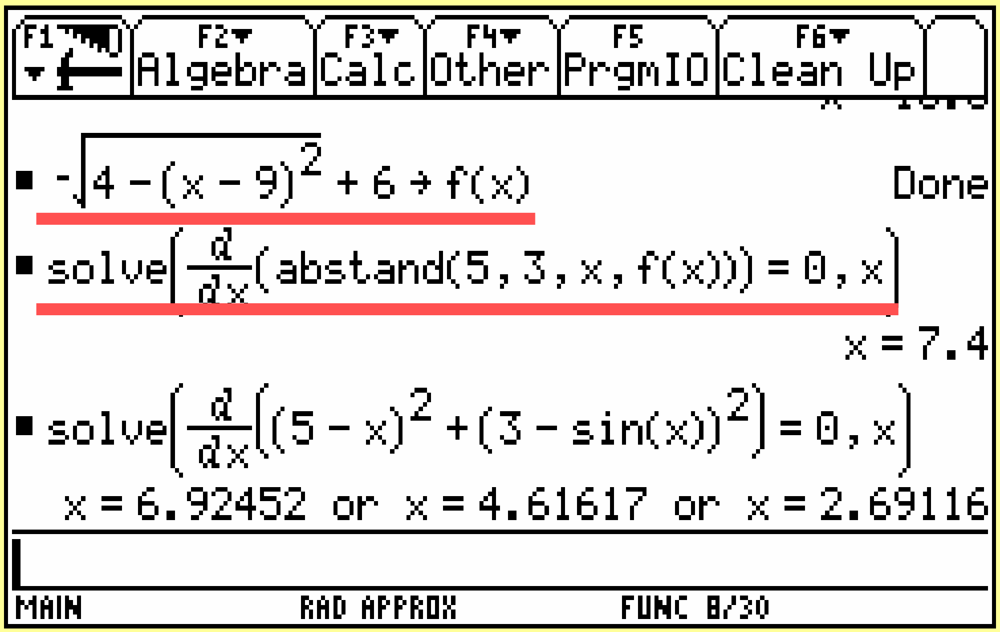


Solution of the x^2-group



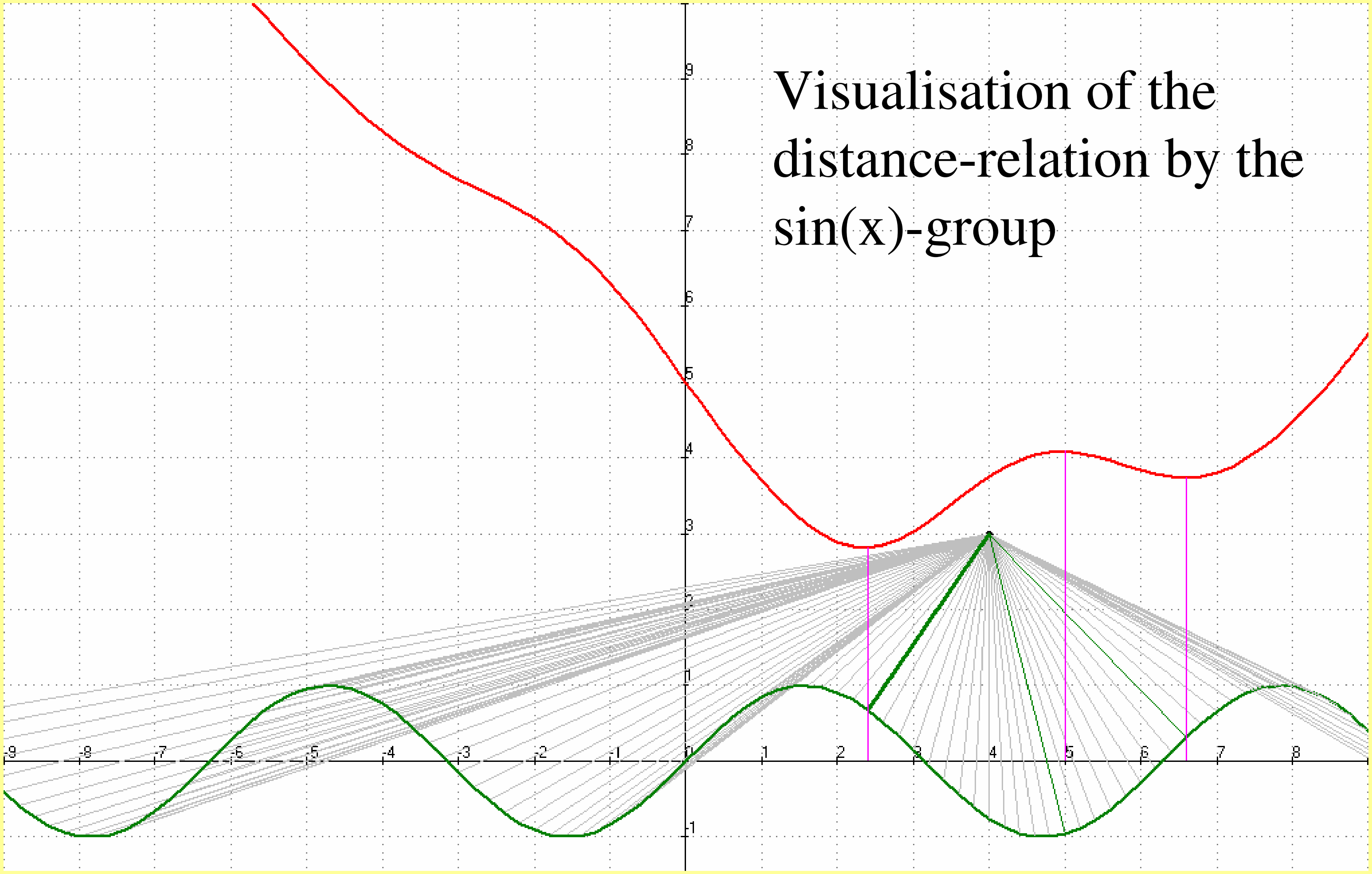
Solution of the sin(x)-group (first trial)





Solution of the circle-group

Hinweis: Die Kreisgruppe hatte sofort erkannt, dass sie ihre Aufgabe ohne Differentialrechnung auf elementare Weise lösen konnte. Eine solche Lösung wurde von der Gruppe auch vorgetragen. Die erhaltenen Ergebnisse wurden erst später durch die hier dokumentierte Bausteinlösung bestätigt.



Abstandsberechnung-sin-mit-Baustein.pl2

A summary of the results

Funktion	D(min) minimaler Abstand	x(min)	y(min)
Sin(x) Trigonometrische Funktion	3.077	6.92452 (4.61617), (2.69116) diese Werte kommen nicht in Frage	0.598
2^x Exponentialfunktion	3.161	2.02	4.059
$x = 2\cos(x) + 9$ $y = 2\sin(x) + 6$ Kreis	3	7.4 (10.6)	4.8
(x+1) / (x+4) Gebrochen-rationale Funktion	2.332	5.0847, rechter Ast, (–4.76662), *x*-Wert für den kürzesten Abstand zum linken Ast	0.6698

The general way of solution

Prinzipiell können alle Abstandaufgaben der Art "kürzester Abstand Punkt *P(a,b)* zum Graphen von $y = f(x)$" mit den folgenden Bausteinen bearbeitet werden:

(1) SQRT((a-c)^2 + (b-d)^2) → abstand(a,b,c,d)

(2) Funktionsterm → f(x)

(3) SOLVE(d/dx(abstand(a,b,x,f(x)), x) = 0, x)

Dabei müssen Sonderfälle beachtet werden.

Diese Sonderfälle betreffen z. B. die Lage des Punktes *P(a,b)* und die Art der Funktionen bzw. Relationen. Auch lässt sich die Lösung manchmal auch elementarer finden.

Das Bausteindreieck

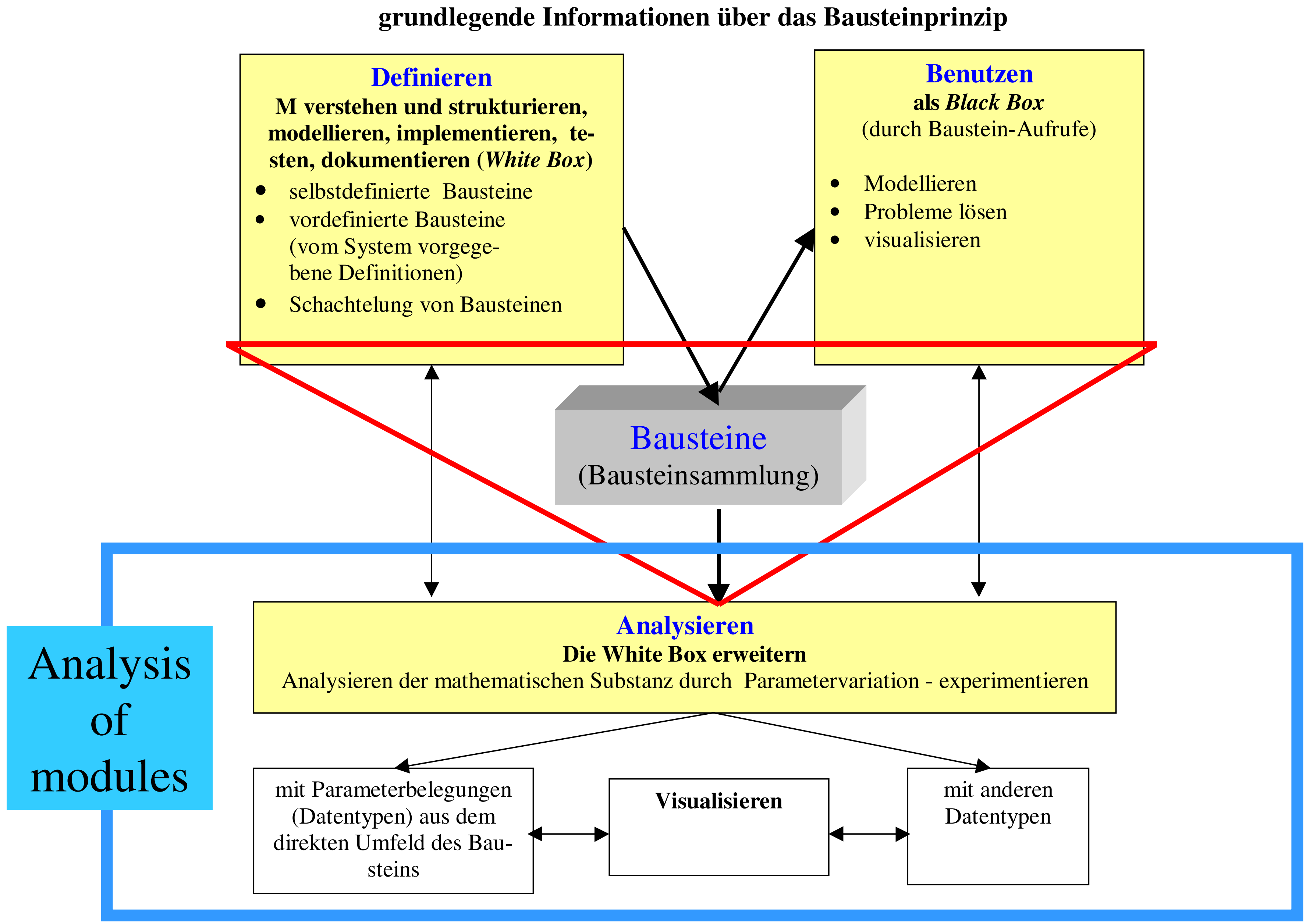


Abb. Das Bausteindreieck: Definieren, Benutzen, Analysieren

Analysis of the distance-module!

sqrt((xa-xb)^2+(ya-yb)^2) → abstand(xa, ya, xb, yb)

abstand(1,4,3,7)

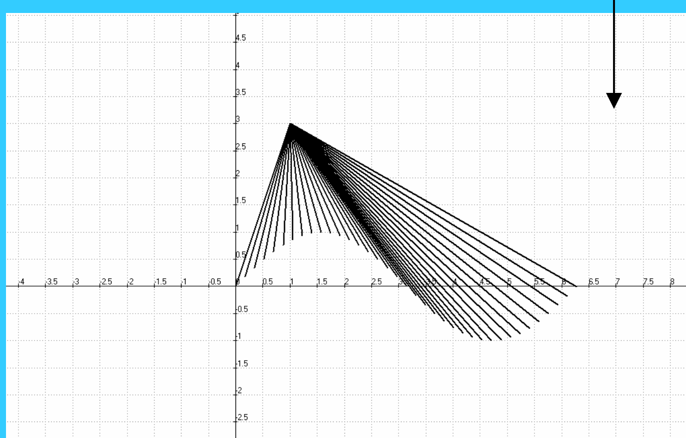
abstand(x, 2x+1, x, sin(x))

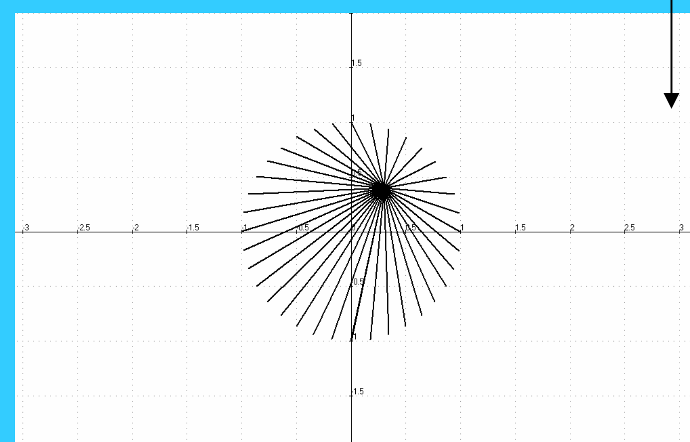
abstand(1,3, x, 2x+1)

abstand(x, cos(x), x, sin(x))

abstand(1,3, x, sin(x))

abstand(0.3, 0.4, sin(x),cos(x))





Solve problems by module-link-up

SOLVE(d/dx(**abstand(a,b,x,f(x)),** x) = 0, x)

Define binobau(a,b,n) = $(a+b)^n$

Binobau(1,1,10)
Zahlenrechnen
Potenzrechnung

Expand(binobau(a,b,2))
Expand(binobau(a,-b,2))
1. und 2. binomische Formel

Expand(binobau(a,b,n) | n={2,3,4,5}
binomische Formeln / Pascalsches Dreieck

binobau(x,1,2)
Funktionen im R2

binobau(x, sin(x), 2)

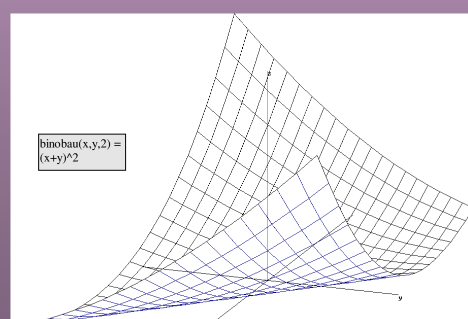
binobau(x,y,2)
Flächen im Raum

binobau(c+d*i*,0,2)
binobau(2+*i*, *i*, 2)
Rechnen mit komplexen Zahlen

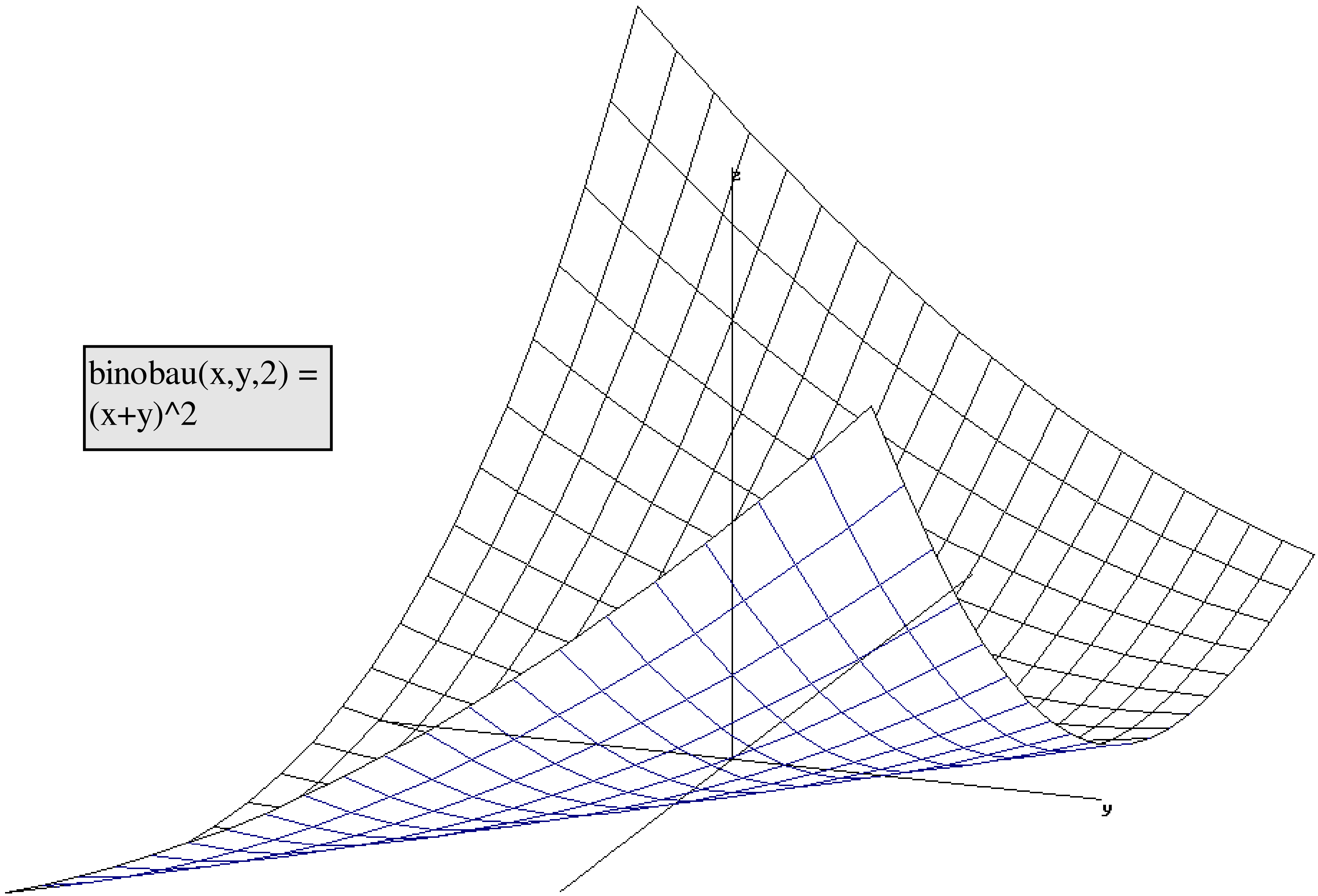
binobau(mata,matb,2)
Rechnen mit quadrat. Matrizen

binobau (binobau (a,1,2),b, 3)
Schachteln von Bausteinen
$((a+1)^2+b)^3$

Analysis of the binom-module



binobau(x,y,2) =
(x+y)^2



Analysis of the module

„Difference quotient"

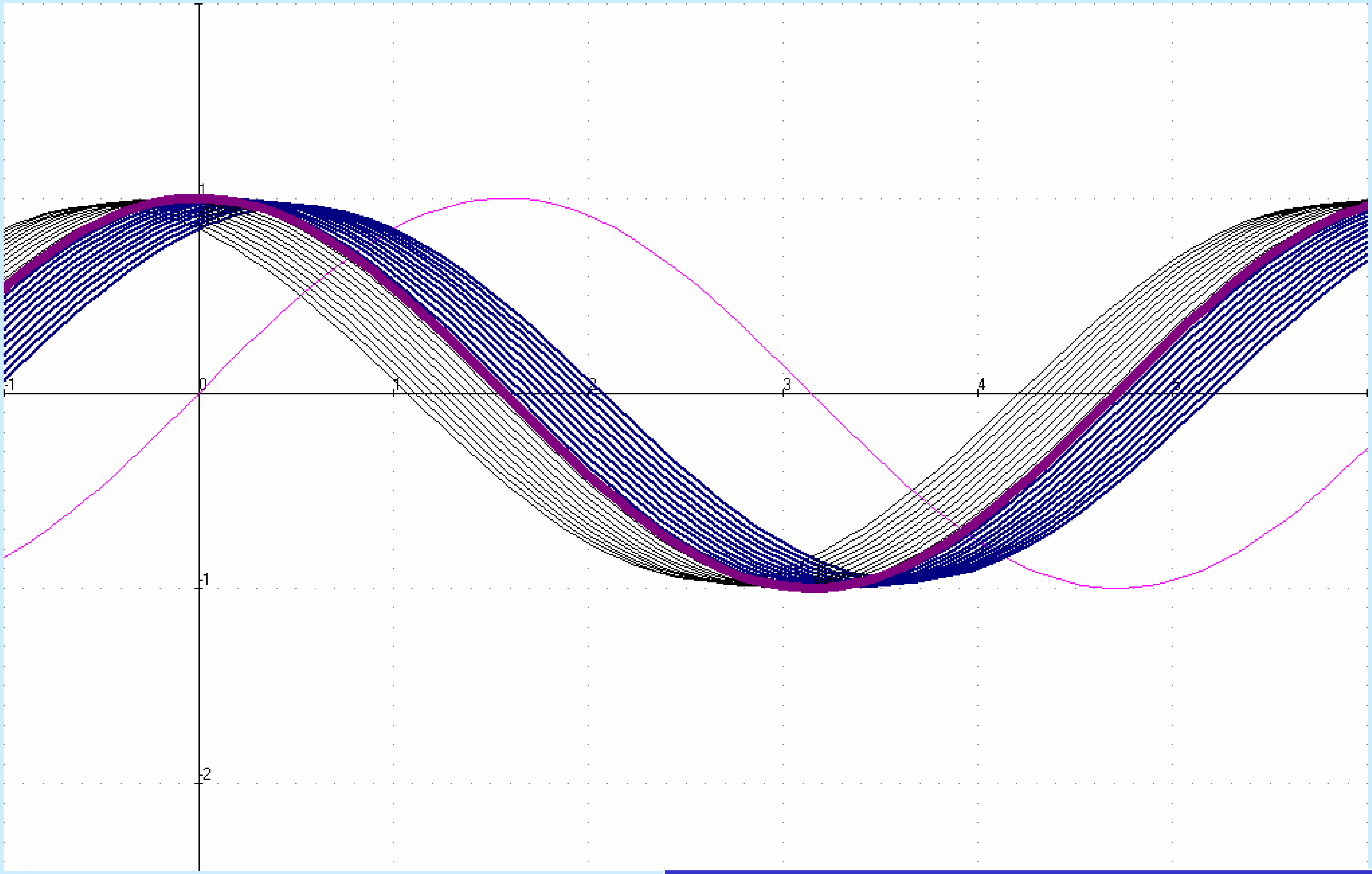
(f(x+h)-f(x)) / h → **dq(x,h)**

for example sin(x)→f(x)

dq(x,h)

this is the function

dq(x,h) = (sin(x+h)-sin(x)) / h



ANIMATO: Diffq-sin-links-rechts.pl2

Analysis of the module

Two main applications

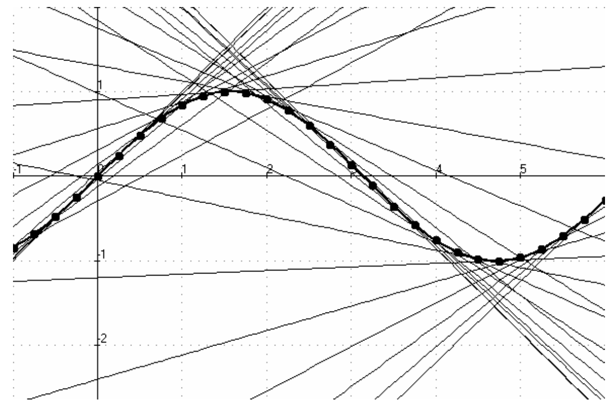
(f(x+h)-f(x)) / h → **dq(x,h)**

for example sin(x)→f(x)

dq(x,h)

This is the function

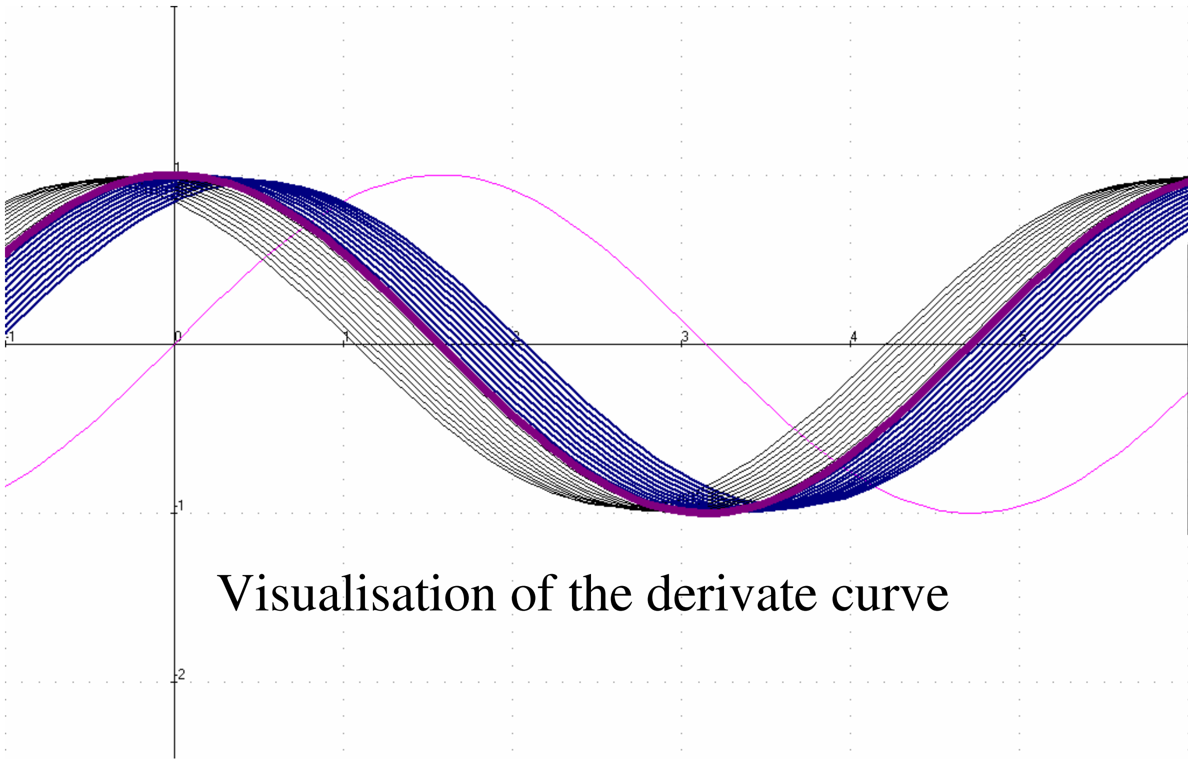
dq(x,h) = (sin(x+h)-sin(x)) / h



wrapped curve

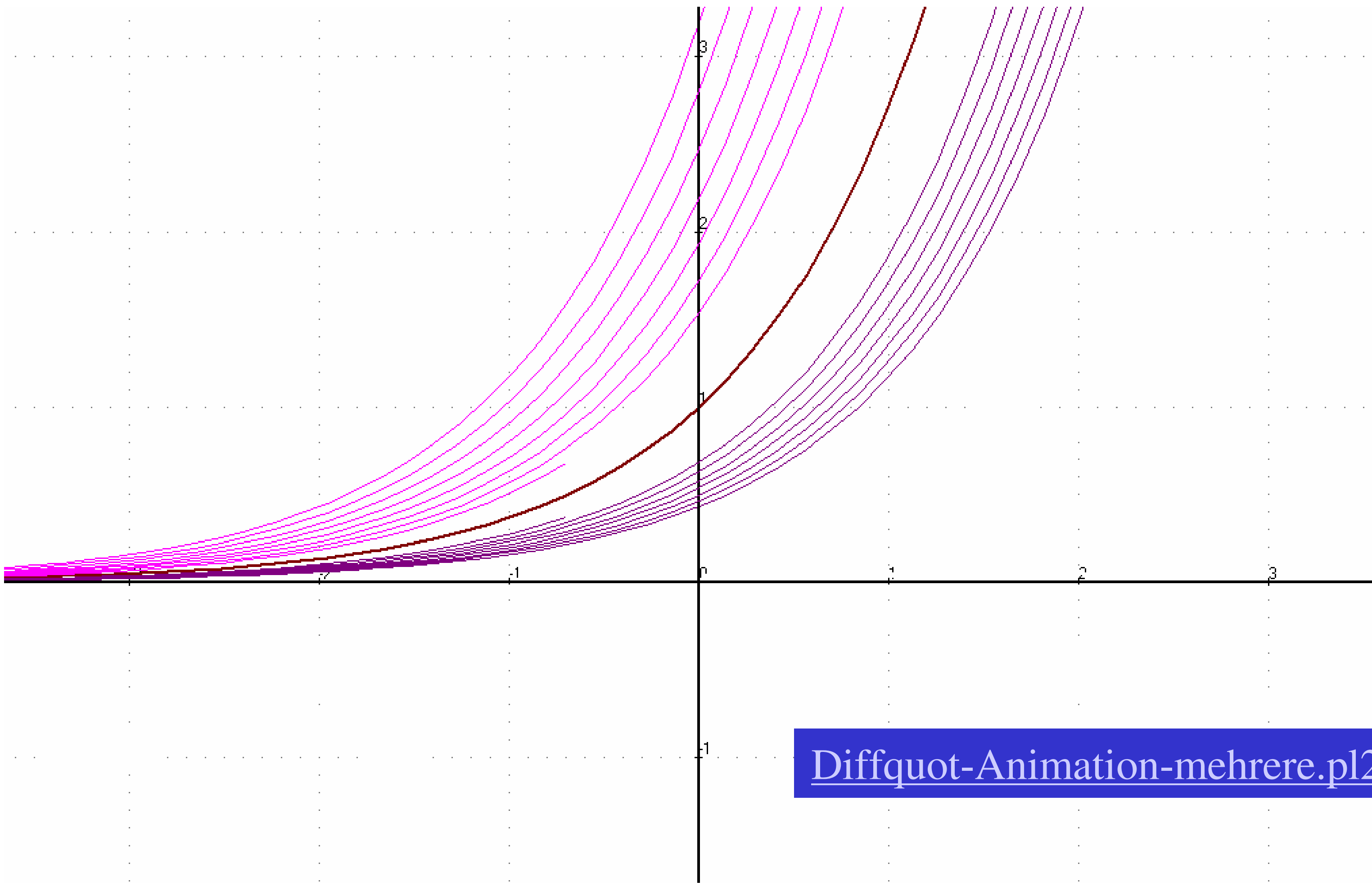
line(x,dq(u,0.01),u,sin(u))

Lines between the points
(u,sin(u)) and (u+1,sin(u+1))
step h=1 **many tangents**



Visualisation of the derivate curve

Animation des Übergangs vom Differenzenquotienten zur Ableitung



Diffquot-Animation-mehrere.pl2

Differenzen-Quotient-Demo

Es wird für mehrere Funktion die Annäherung an die Ableitungsfunktion demonstriert

f2: sin(a)
f3: (f2(t+u)-f2(t))/u
f4: (f2(t-u)-f2(t))/(-u)
f5: cos(t)

f7: e^a
f8: (f7(t+u)-f7(t))/u
f9: (f7(t-u)-f7(t))/(-u)
f10: e^t

f12: a^3
f13: (f12(t+u)-f12(t))/u
f14: (f12(t-u)-f12(t))/(-u)
f15: 3t^2

f17: int(x),0 Nachziehen der Achsen
f18: 0,int(x)

Die dahinter stehende Mathematik

Further examples B (short)

Modules of modules

VECTOR(dif(SIN(x)^n, x), n, 1, 6, 1)

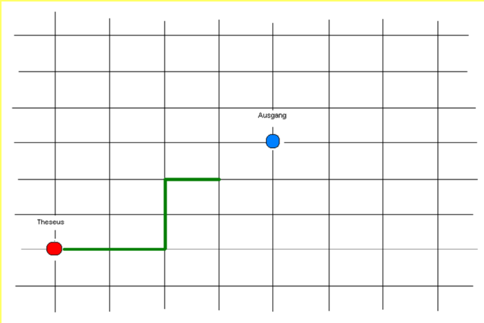
[COS(x), 2·SIN(x)·COS(x), 3·SIN(x)^2·COS(x), 4·SIN(x)^3·COS(x), 5·SIN(x)^4·COS(x), 6·SIN(x)^5·COS(x)]

Modules of modules (modules link up)

Eine Folge von TI- Befehlen (statt eines Programms)

E.Lehmann, aus einem Schülerreferat , LK von Cordula Kollotschek

„Damit die Flucht gelingen kann, muss Theseus insgesamt 4 Mal nach rechts und 3 Mal nach oben abbiegen. - Statt „ Kopf “ und „ Zahl “ werden wir mit dem TI die Zahlen 0 und 1 als mögliche Ergebnisse eines Münzwurfs benutzen. Für unser Beispiel soll 1 bedeuten, dass in x-Richtung und 0, dass in y-Richtung gegangen wird.“



- Rand(2)-1
- Seq(rand(2)-1,i,1,7)
- Sum(Seq(rand(2)-1,i,1,7))
- Sum(Seq(rand(2)-1,i,1,7))→einsen(i)
- When(einsen(i)=4,1,0)→erfolg (i)
- Sum(Seq(erfolg (i),i,1,100))

- Sum(Seq(when(sum(Seq(rand(2)-1,i,1,7)) = 4,1,0), i, 1, 100)

„Wer schreibfaul ist und alles in einem Schritt eingeben möchte, braucht nur diesen Befehl einzugeben und es werden alle Befehle auf einmal ausgeführt!“

A magic-square-module

Let a, b, c independent variables, then

the algebraic for a magic square becomes

mat (a,b,c) :=

c-b	c+a+b	c-a
c-a+b	c	c+a-b
c+a	c-a-b	c+b

mat(3,1,4)

3	8	1
2	4	6
7	0	5

Experiences with the magic-square-module

mat(a, b, c) ≔ [c - b, a + b + c, c - a; -a + b + c, c, a - b + c; a + c, -a - b + c, b + c]

VECTOR(mat(a, b, c)^k, k, 1, 5)·[1; 1; 1] **What´s about the matrice-powers?**

Are they magic too?

mat^1 [[3·c, 3·c, 3·c]; sums of the rows of matrice-power

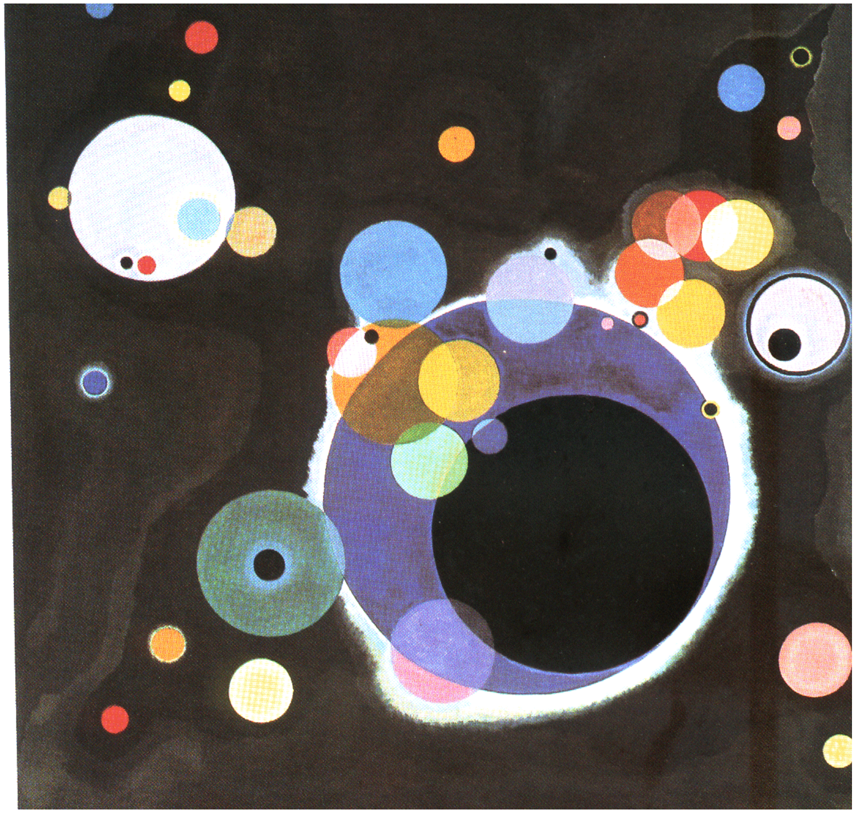
mat^2 [9·c^2, 9·c^2, 9·c^2]; do the same for the columns

mat^3 [27·c^3, 27·c^3, 27·c^3]; do the same for the diagonales

mat^4 [81·c^4, 81·c^4, 81·c^4];

mat^5 [243·c^5, 243·c^5, 243·c^5]]

Kandinskybild als Hintergrund

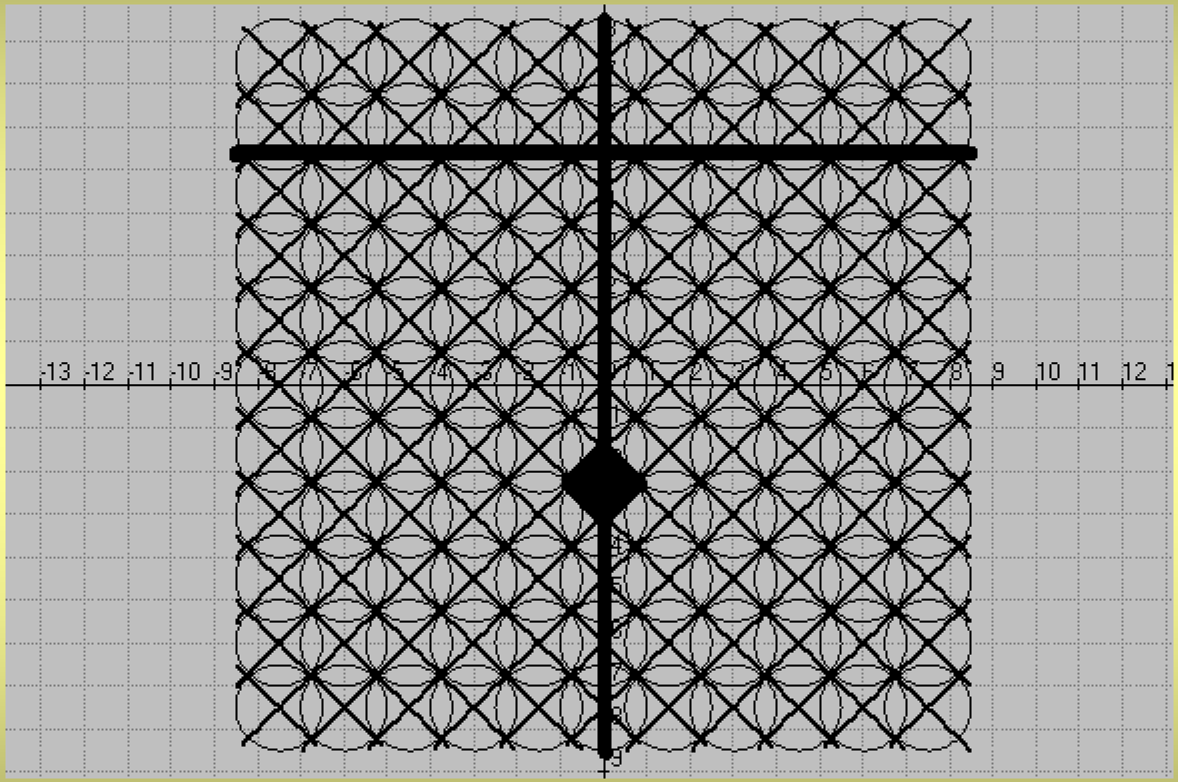


Kandinsky-Kreise-Bausteine.pl2

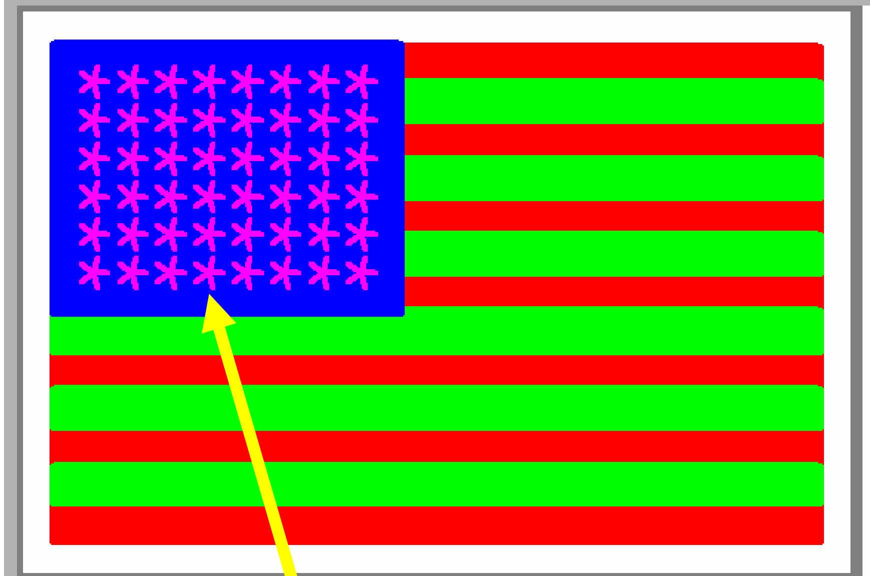
Die dahinter stehende Mathematik

f1: t	
f2: sin(t)+3.5, cos(t)-3	ein Kreis
f3: 3.4*sin(t)+3.5,3.4*cos(t)-3	noch ein Kreis
f4: v*sin(t)+3.5,v*cos(t)-3	Kreisscheibe
f6: a*cos(b*t)+c	**circle-module**
f7: a*sin(b*t)+c	
f8: f6(3.4,1,3.5),f7(3.4,1,-3)	Bausteinaufruf
f9: -6.2,5.2	ein Punkt
f10: f6(v,1,-6.2),f7(v,1,5.2)	Bausteinaufruf

Some problems – solved with modules:



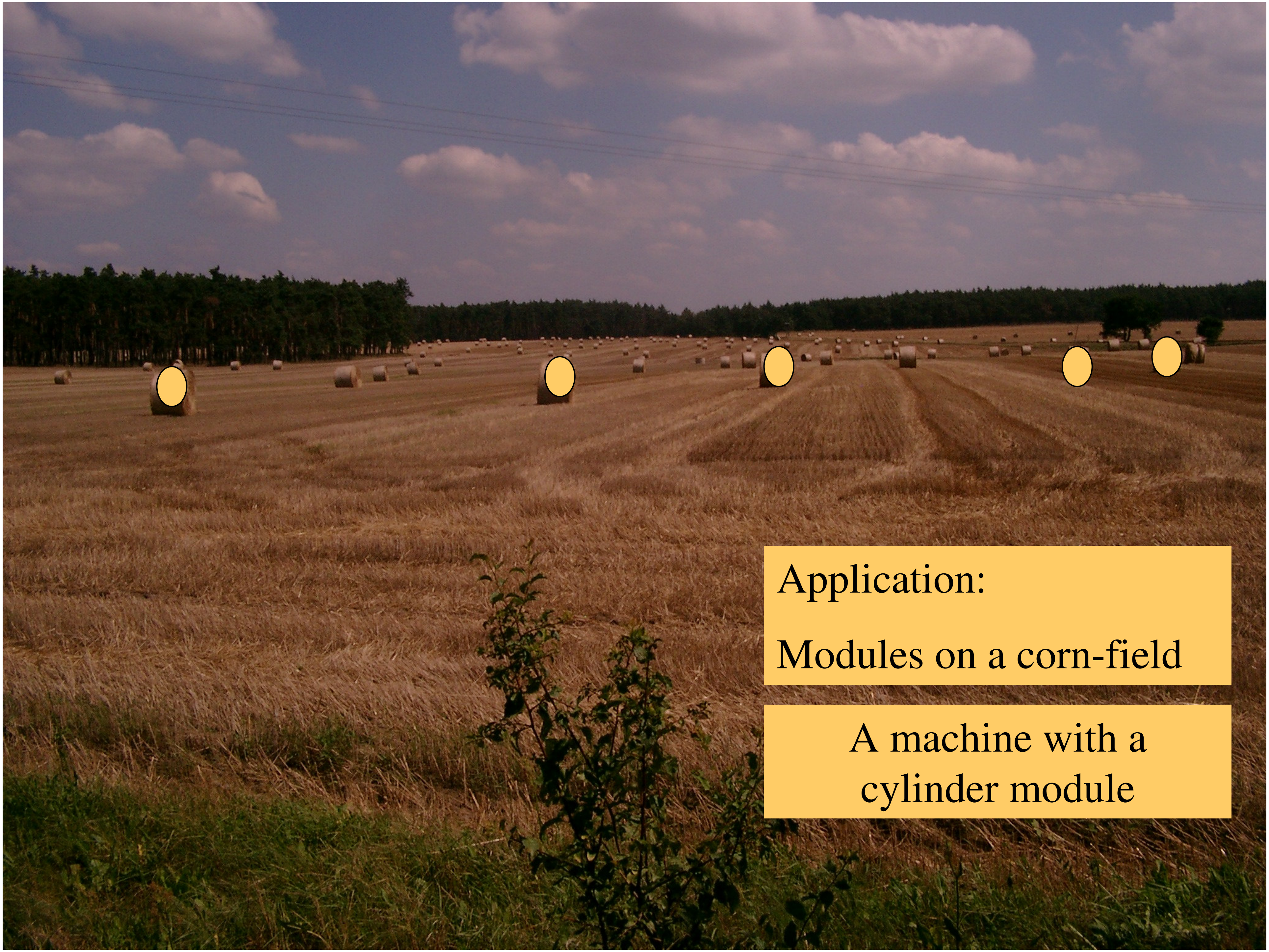
Türgitter - fotografiert in Kalabrien

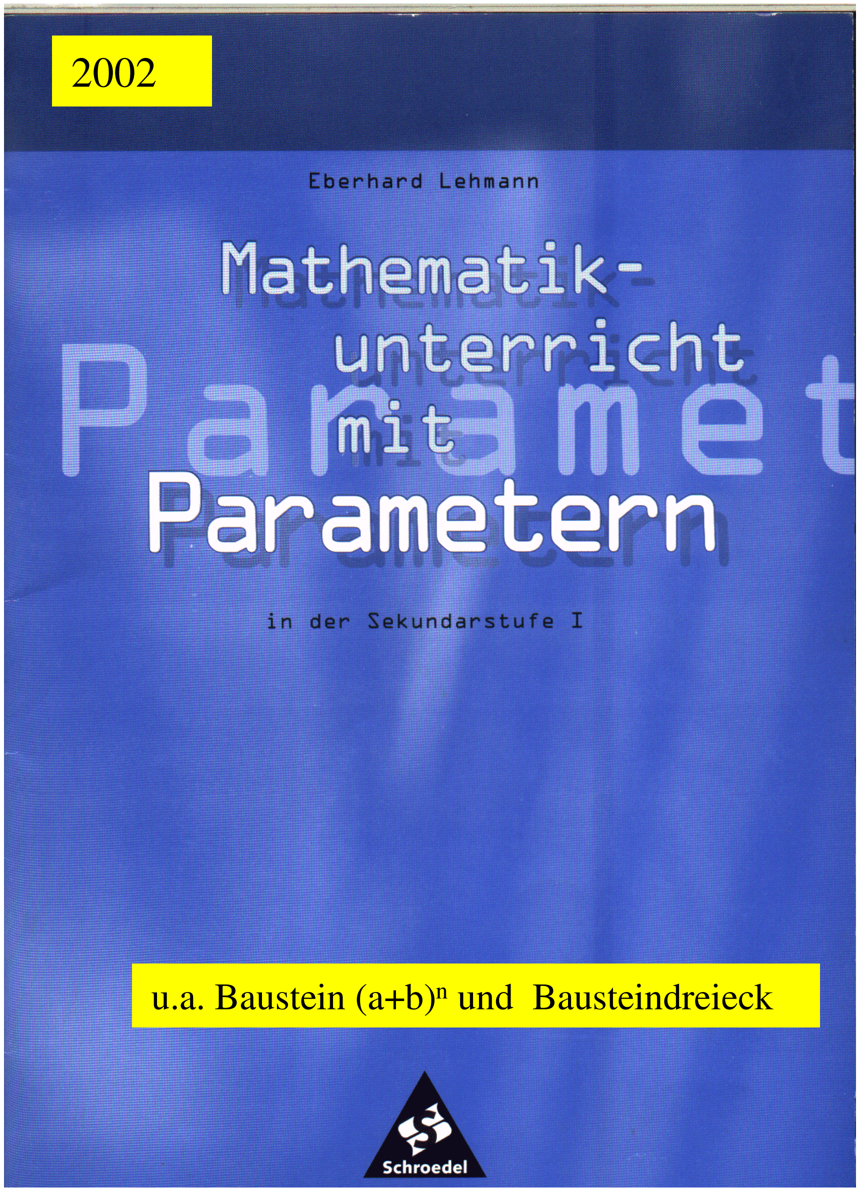


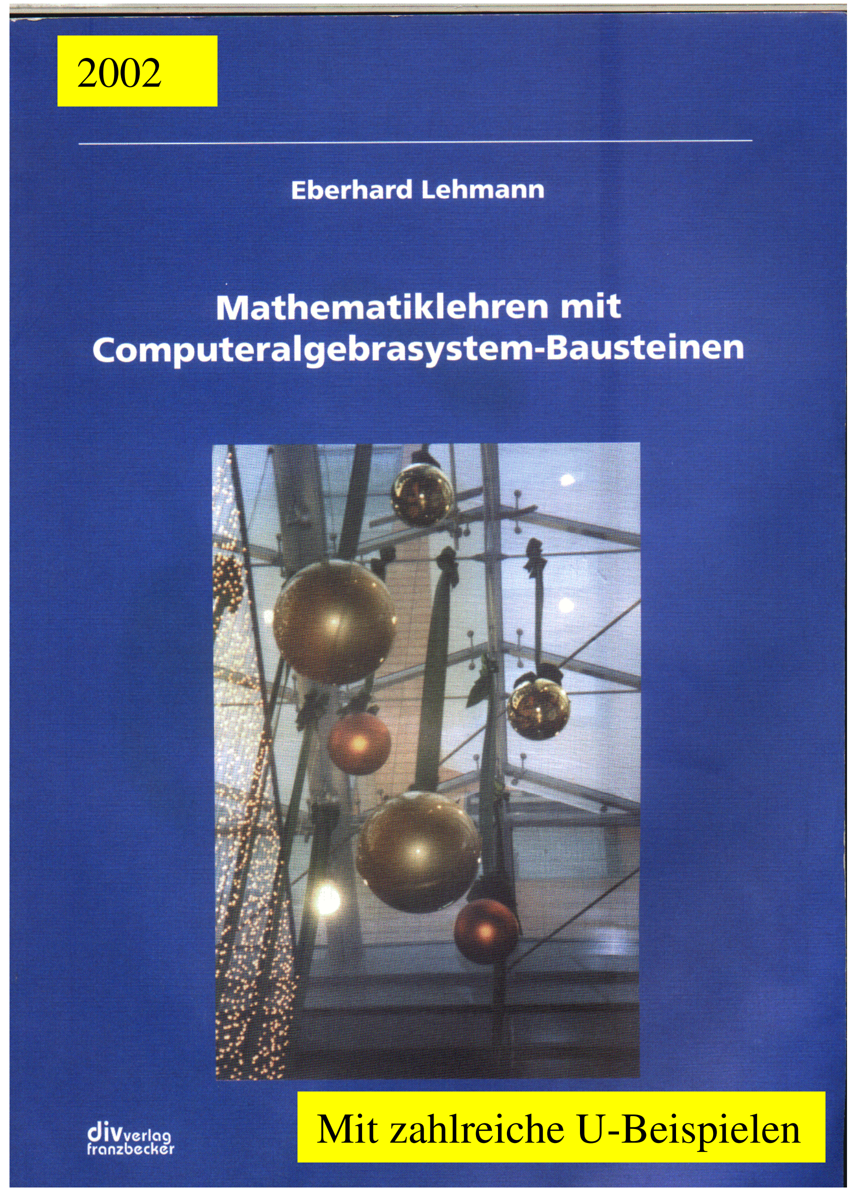
Jasper Johns-2.pl2

Das ruft nach Parametern!

Plot2-oetz2.exe


Application:
Modules on a corn-field
A machine with a
cylinder module





And some essays in magazines PM, MNU, MU

Opinions of my students concerning the use of modules

With a module you can solve a wide field of problems much faster.

Using modules you have a look at all variables.

Investigate the effect of a parameter!

Solve problems by changing the values.

With modules we can explore in a wonderful way.

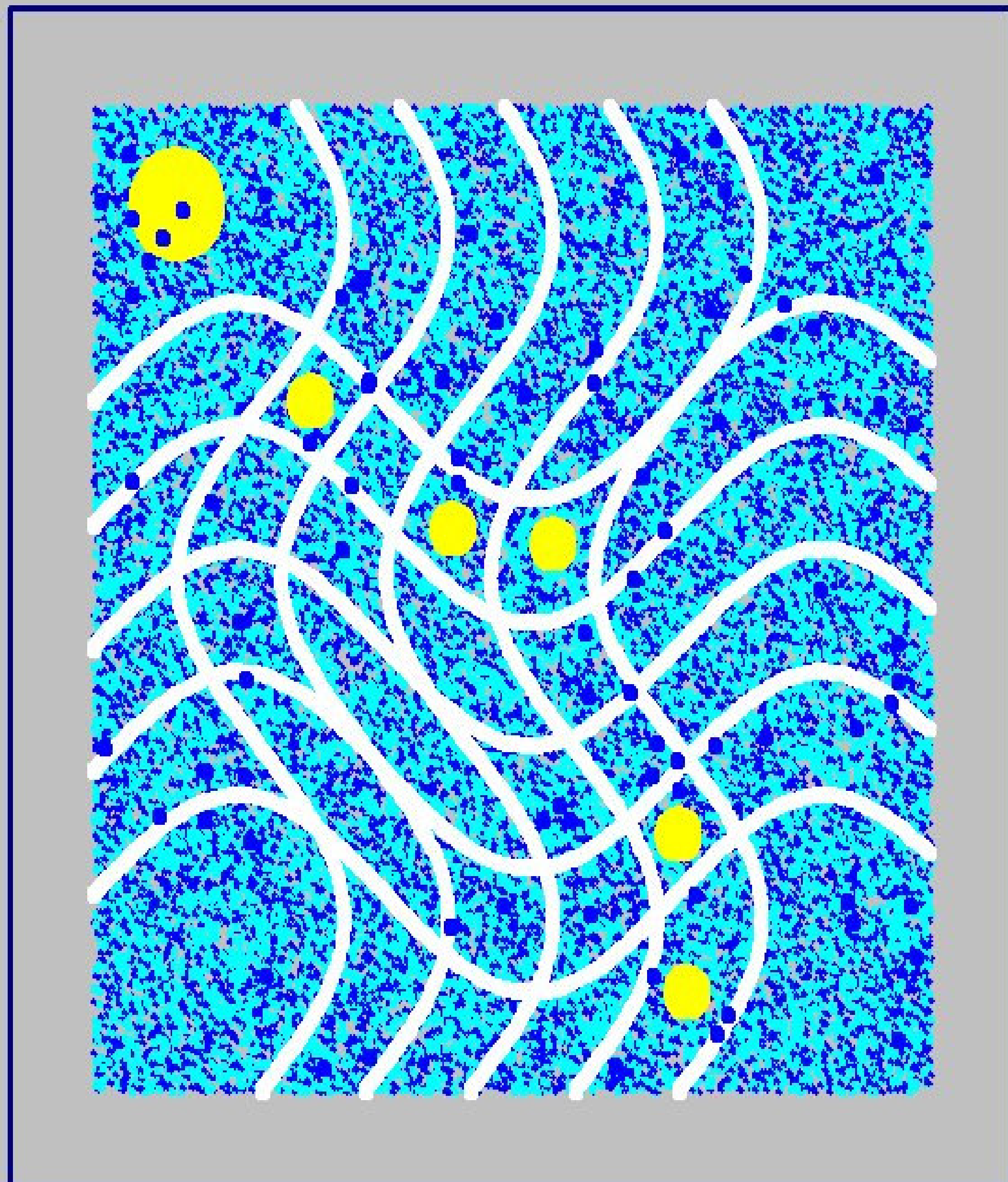
Modules are tools to compress a mathematical field.

The module-principle favors the generalisation of calculations and solutions.

There are many software-systems were you can work with tools like procedures and modules and their parameters. This is important to find economic and elegant solutions for your problems.

Warning! Don´t loose the look on the mathematical term.

STOP



I hope you are convinced from

the power of modules

Use the system-modules or your own modules to simplify your work!

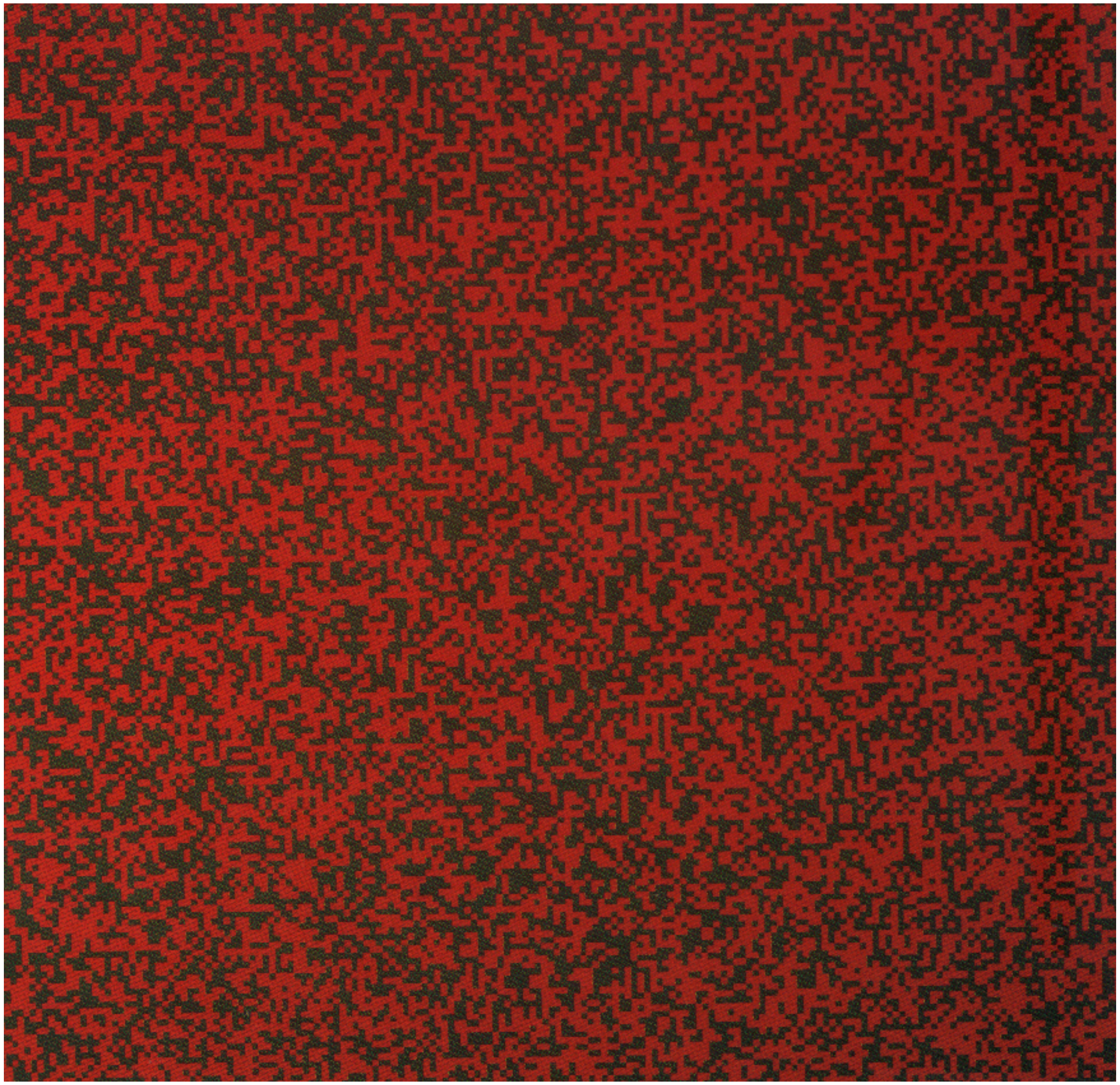
Analyse modules for better understanding of mathematic!

Use modules to find general solutions!

The users of computers need modular competence!

Thank you! – Dr. Eberhard Lehmann, Berlin 2006

Francois Morellet: A square of random-numbers



random

random