Propositional Attitudes

Examples

Relational Belief

Referential Opacity

Propositional Attitudes and Referential Opacity - p. 1: 104

Introduction to the Philosophy of Language

Winter 2004

Erich Rast
http://akira.ruc.dk/~erast/
Roskilde University
erast@ruc.dk

Propositional Attitudes

“Propositional Attitudes and Referential Opacity

To understand a proposition means to know what is the case, if it is true. One can therefore understand it without knowing whether it is true or not. One understands it if one understands its constituent parts.”

(Wittgenstein, Tractatus Logico-Philosophicus, 4.024)

What are Propositional Attitudes?

Here are some examples:

(1) Bebel fancies that the return of Alsace-Lorraine would appease France’s desire for revenge. (Frege 1892, 48)
(2) Ralph believes that Orcutt is a spy. (Quine 1956, 180)
(3) John fears darkness.
(4) Lois Lane believes that Superman can fly.
(5) Lois Lane knows that Superman doesn’t exist.
(6) John thinks that Mary loves Peter.
(7) Lois Lane refuses to accept that Superman doesn’t exist.

Examples 2 & 4 involving “to believe” are canonical examples.
The other examples are more problematic or controversial.
Why the term “propositional attitudes”?
The basic idea of propositional attitudes: The subject is in a certain attitude relation to the proposition expressed by the embedded sentence.
Questions: What kinds of relations? What is a proposition?

Literature

Quine (1956): Quantifiers and Propositional Attitudes

Kripke (1979): A Puzzle about Belief.
Relational Belief

(8) Peter believes that Mary loves John.

\[ \text{Peter believes (Mary loves John)} \]

- Peter is in the belief relation to the proposition that is expressed by the embedded sentence «Mary loves John».
- The relation is between an actual and an abstract object.
- The meaning of a complete sentence is often called proposition.
- Frege claimed that the odd Fregean meaning of a sentence was its sense, not its truth value.
- A proposition is just the same as the Fregean sense of a sentence, except that it is often thought of in a more technical way, for example as something that is possible.

What is Referential Opacity?

(9) Lois Lane believes that Superman can fly.
(10) Lois Lane doesn’t believe that Clark Kent can fly.
(11) Superman can fly.

\[ \text{Lois believes (Superman can fly)} \]

- As we all know, Superman is Clark Kent.
- So «Superman» and «Clark Kent» are co-referential.
- We can substitute «Clark Kent» for «Superman» in 11 salva veritate (=without changing the truth value).
- But we cannot substitute «Clark Kent» with «Superman» in 9 or 10 salva veritate.
- The belief ascription is referentially opaque.
- More precisely, the reading of the ascription is opaque. It can also be read as being referentially transparent, allowing substitution.
- Sometimes the opaque reading is called de dicto and the transparent reading de re.

Referential Opacity

More Examples

(12) Erich Rast believes that Ruth Barcan found the Barcan Formula.
(13) Erich Rast doesn’t believe that Ruth Marcus found the Barcan Formula.
(14) Ruth Barcan found the Barcan formula.
(15) Ruth Marcus found the Barcan formula.

\[ \text{Erich Rast believes (Ruth Barcan found the Barcan Formula)} \]

- It doesn’t matter whether the referent is fictional or real. Referential opacity can arise with actual referents.
- Question: Can two embedded sentences with co-referential proper names express different propositions?
- Answer: That depends on what propositions are supposed to be.
- Moral: You should avoid talk about propositions without giving them an exact definition or exact identity conditions.
Views on Referential Opacity

16) Lois Lane believes that Superman can fly.
17) Lois Lane believes that Clark Kent can’t fly.

Two main positions:

- **Semantic View**
  Two attitude ascriptions with an embedded proper name can have different truth values if the proper names are co-referential.

- **Pragmatic View**
  Two attitude ascriptions with an embedded proper name have the same truth values, if the proper names are co-referential, but may differ pragmatically, for example in their cognitive value for speakers/hearers.

The semantic position is common, whereas the pragmatic view has rarely been propagated (e.g. by Salmon (1986); McKay (1981)). If referential opacity is considered a pragmatic phenomenon, it poses no problems for a semantic analysis. How could it? But of course the problem will reappear once you try to get a formal grasp on the pragmatics of belief ascriptions.

Propositional Attitudes and Modal Logic

Modal logics are often used for modelling propositional attitudes.

18) Ralph believes that Orcutt is a spy.

**Simplified:**

\[
\text{believe}(x, y) \quad \{w|\text{Orcutt is a spy in } w\}
\]

*the set of all worlds in which Orcutt is a spy*

How it really works:

19) \( M, g, w \models B_{w} P(b) \)

\[
\Downarrow
\]

20) In all \( w' \) such that \( wR_{w} w' : M, g, w' \models P(b) \)

\[
\Downarrow
\]

21) In all \( w' \) such that \( wR_{w} w' : T_{g}(b) \in I(P, w') \)

Attitudes in Modal Logic

Recall: A model \( M = (W, R, D, I) \) is a Kripke model for a first-order modal logic consisting of a set of possible worlds \( W \), an accessibility relation \( R \), the domain \( D \), and interpretation function \( I \).

A multi-modal Kripke model is just the same, except that \( R \) is a set of binary relations \( R_{i} \) between possible worlds, each of those relations with an index \( i \).

So \( R_{w} \) can be a relation between two worlds \( w, w' \in W \) such that

- \( w \) is the world we are evaluating at, e.g. the actual world
- \( w' \) is a world that is compatible with what Ralph (symbolized by the index \( o \)) believes in \( w \).

- \( B_{w} \) is an operator like \( \Box \).
- So \( M, g, w \models B_{w} P(b) \) informally means something like \( P(b) \) is true in all worlds that are compatible with what Ralph believes in \( w \).

\( [i] \) and \( (i) \) are also often used as a notation for operators corresponding to \( \Box \) and \( \Diamond \) with accessibility relation \( R_{i} \) in multi-modal systems. We’ll stick to \( B_{w} \) for belief and \( \Box \) for alethic modality.

Limitations: We have a fixed set of agents represented as indices. By default, it’s not possible to quantify over agents, as this would require us to perform advanced set theoretic operations over various relations \( R_{i} \), \( R_{j}, \ldots \).
**A Problem With Doxastic Modalities**

In normal modal logic, referential opacity cannot be encoded using ordinary constants. Ordinary constants are rigid with respect to any modality. Suppose $M = (W, R, D, I)$ such that $I(c, w) = I(c, w')$ for any constant $c$ and $w, w' \in W$. Let $a, b$ be constants that denote the same object, i.e. $I(a, w) = I(b, w)$.

(22) $M, g, w \vDash BP(a)$

$\Box$ rule for belief operator

(23) in all $w'$ such that $wRw'$: $M, g, w' \vDash P(a)$

$\Box$ rules for predicates and constants

(24) $I(a, w') \in I(P, w')$

$\Box$ by rigidity

(25) $I(a, w) \in I(P, w')$

because $I(a, w) = I(b, w)$

(26) $I(b, w) \in I(P, w')$

$\Box$ like above, but in reverse direction

(27) $M, g, w \vDash BP(b)$

---

**The Underlying Problem**

- The proposed solution only works if $R_\Box$ doesn't grant access to all worlds that are compatible with any agent's belief worlds.
- In other words, there are some worlds that are reachable by some doxastic modality $R_\Box$ but not by the $R_\Box$ relation.
- Formally: $\neg \forall w_0, w_3 \left( w_0 R_\Box w_1 \land w_0 R_\Box w_2 \rightarrow w_3 R_\Box w_4 \right)$.
- Let's call the set of worlds that are reachable by a doxastic accessibility relation $W_\Box$ and the set of worlds that are reachable by an alethic accessibility relation $W_\delta$.
- Then the above condition implies that $W_\Box$ is not a subset of $W_\delta$.
- So some belief worlds are elements in $W_\Box$ but not in $W_\delta$. Example: $w_0$ on the last slide.
- But then, whatever statements this world makes true are impossibly true. ($\Box \phi$ is true in any such world.)
- This is not very plausible.
- At least it's not desirable for solving referential opacity, since it is unreasonable to claim that an agent believes something impossible when he just doesn't know that two names are co-referential.

---

**A Possible Solution?**

Co-referential constants are substitutable within the scope of doxastic modalities because they are rigid no matter which kind of modality is involved. Make them non-rigid in respect to doxastic modalities and the problem seems to be solved.

We can, it seems, still define rigidity for alethic modalities:

- Rigidity in respect of $\Box$. For any constant $c$, any $w_0$, and all $w_1, w_2$ such that $w_0 R_\Box w_1$ and $w_0 R_\Box w_2$: $I(c, w_1) = I(c, w_2)$.

- Non-rigidity in respect of $B$. For any constant $c$, any $w_0$, and doxastic modality $\Box$, there can be two worlds $w_1, w_2$ such that $w_0 R_\Box w_1$ and $w_0 R_\Box w_2$ and $I(c, w_1) \neq I(c, w_2)$.

An example:

$w_0 R_\Box w_0 \quad I(a, w_0) = a$

$w_0 R_\Box w_1 \quad I(a, w_1) = a$

$w_0 R_\Box w_2 \quad I(a, w_2) = a$

$w_0 R_\Box w_3 \quad I(a, w_3) = a$

$w_0 R_\Box w_4 \quad I(a, w_4) = b$

- Why doesn't this work?

---

**More Problems**

**Overgeneration**

As a consequence of the following properties of normal modal logics more formulas are valid than might be desired.

**Omniscience**

If $A$ is provable, then it is also known or believed. So every logical tautology is automatically believed. The agent is logically omniscient. Necessitation rule: if $\vdash A$ then $\Box A$.

**Distribution of $\land$ and $\rightarrow$**

If an agent believes/knows that $A \land B$, he also believes/knows that $A$ and believes/knows that $B$. This is sometimes considered inplausible. In K

$\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$ (K-Rule) and

$\Box(p \land q) \rightarrow (\Box p \land \Box q)$ are valid, but not the disjunctive version.
**Even More Problems**

**Undergeneration**
As a result of the following properties of normal modal logics, less formulas are valid than might be desired.

**Substitutability of Rigid Constants**
In a standard treatment, constants are rigid and therefore substitutable with co-referential constants in any context. Making terms non-rigid in respect to one modality but rigid with respect to another is difficult, not standard, and not trivial.

**Inner-world Substitutability**
In a normal first-order modal logic, singular terms that are co-referential in one world are substitutable for each other in relation to this world. This has been considered problematic in connection with essential indexicals. If 

\[ T_g(t_1, w) = T_g(t_2, w), \]  

then \( M, g, w \models A \iff A[t_1/t_2] \).

---

**Formal Treatments of Propositional Attitudes**

The approaches to deal with propositional attitudes can be classified according to several criterias.

- **Syntactic Approach**: quote a formula or linguistic entities themselves
- **Direct Quotation**: use a quoting operator in the formal language itself: \( D(w, 'P(b)') \) “Quine-corners”. In combination with strong reflection principles, inconsistencies can arise (Montague’s theorem), but there are ways to deal with them. Lit. Carnap (1947), Davidson (1969), Wessel (1998), Bolander (2003)
- **Indirect Quotation**: refer to linguistic material in another language, e.g. the natural language that is analysed, or syntactic material that is represented in parallel with the actual formulas. Lit. Token-dependency Semantics, Dahlsp (2002)
- **Non-syntactic Approach**: modify modal logics, use other logics

**Kripke’s Puzzle**

Living in France, Pierre comes to believe that London is pretty. Later, when he is living in London, he comes to believe that London is ugly, but doesn’t realize that “Londres” and “London” denote the same city.

(28) Pierre: Londres est jolie  
   → Pierre believes that London is pretty.

(29) Pierre: London is ugly.  
   → Pierre believes that London is ugly.

**Disquotation**

The puzzle: We conclude that Pierre has contradictory beliefs, yet he seems to be a perfectly rational agent. Pierre hasn’t changed his beliefs. Does he believe that London is pretty or doesn’t he believe that London is pretty?
**Principle of Disquotation**

For an English sentence p:

"If a normal English speaker, on reflection, sincerely asserts to 'p', then he believes that p." (Kripke 1979, 112/113)

- The principle is language-dependend.
- The principle assumes that the speaker is sincere.
- The principle assumes that the speaker reflects what he says. (He is not just babbling like an idiot or parrot.)
- Kripke's puzzle assumes two disquotation principles, one for French and one for English.

**Principle of Translation**

For languages:

"If a sentence of one language expresses a truth in that language, then any translation of it into any other language also expresses a truth (in that other language)." (Kripke 1979, 112/113)

- The principle assumes that it is possible to translate from one to another language.
- This may be critizised, but this critique would be external to the argument.
- Philosophical quiddities aside, «Londres est jolie» and «London is pretty» are clearly translations of each other.
- Question: Does this principle also apply to believe sentences with referentially opaque reading?

**Some Questions About Belief Ascriptions**

- What are the criteria for...
- ...truth or falsity of belief ascriptions?
- ...acceptance or rejection of belief ascriptions
  - ...by the linguistic community?
  - ...by the speaker?
  - ...by the believer?
- In what way does a sentence express an attitude of the speaker?