Introduction to the Philosophy of Language

Winter 2004

Erich Rast

http://akira.ruc.dk/~erast/

Roskilde University
erast@ruc.dk
“To understand a proposition means to know what is the case, if it is true. (One can therefore understand it without knowing whether it is true or not.) One understands it if one understands its constituent parts.”

(Wittgenstein, *Tractatus Logico-Philosophicus*, 4.024)

**Literature**

- Quine (1956): *Quantifiers and Propositional Attitudes*
- Kripke (1979): *A Puzzle about Belief.*
Propositional Attitudes
What are Propositional Attitudes?

Here are some examples:

(1) Bebel fancies that the return of Alsace-Lorraine would appease France’s desire for revenge. (Frege 1892, 48)

(2) Ralph believes that Ortcutt is a spy. (Quine 1956, 180)

(3) John fears darkness.

(4) Lois Lane believes that Superman can fly.

(5) Lois Lane knows that Superman doesn’t exist.

(6) John thinks that Mary loves Peter.

(7) Lois Lane refuses to accept that Superman doesn’t exist.
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- The basic idea of propositional attitudes: The subject is in a certain attitude relation to the proposition expressed by the embedded sentence.
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- Why the term »propositional attitudes«?
- The basic idea of propositional attitudes: The subject is in a certain attitude relation to the proposition expressed by the embedded sentence.
- Questions: What kinds of relations? What is a proposition?
(8) Peter believes that Mary loves John.

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\text{believe}(x, y) \quad \text{Peter} \quad \text{the proposition that Mary loves John}
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- Peter is in the belief relation to the proposition that Mary loves John.

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- Peter is in the belief relation to the proposition that is expressed by the embedded sentence »Mary loves John«.
- The relation is between an actual and an abstract object.
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- Frege claimed that the odd Fregean meaning of a sentence was its sense, not its truth value.
- A proposition is just the same as the Fregean sense of a sentence, except that it is often thought of in a more technical way, for example as a set of possible worlds.
Referential Opacity
What is Referential Opacity?

(9) Lois Lane believes that Superman can fly.
(10) Lois Lane doesn’t believes that Clark Kent can fly.
(11) Superman can fly.

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- But we cannot substitute »Clark Kent« with »Superman« in 9 or 10 \textit{salva veritate}. 
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- More precisely, the reading of the ascription is opaque. It can also be read as being referentially transparent, allowing substitution.
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So »Superman« and »Clark Kent« are co-referential.

We can substitute »Clark Kent« for »Superman« in (11) \textit{salva veritate} (=without changing the truth value).

But we cannot substitute »Clark Kent« with »Superman« in (9) or (10) \textit{salva veritate}.

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Sometimes the opaque reading is called \textit{de dicto} and the transparent reading \textit{de re}.
More Examples

(12) Erich Rast believes that Ruth Barcan found the Barcan Formula.
(13) Erich Rast doesn’t believe that Ruth Marcus found the Barcan Formula.
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- It doesn’t matter whether the referent is fictional or real. Referential opacity can arise with actual referents.
- Question: Can two embedded sentences with co-referential proper names express different propositions?
- Answer: That depends on what propositions are supposed to be.
- Moral: You should avoid talk about propositions without giving them an exact definition or exact identity conditions.
Views on Referential Opacity

(16) Lois Lane believes that Superman can fly.
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Two main positions:
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- **Semantic View**
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The semantic position is common, whereas the pragmatic view has rarely been propagated (e.g. by Salmon (1986); McKay (1981)). If referential opacity is considered a pragmatic phenomenon, it poses no problems for a semantic analysis.
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The semantic position is common, whereas the pragmatic view has rarely been propagated (e.g. by Salmon (1986); McKay (1981)). If referential opacity is considered a pragmatic phenomenon, it poses no problems for a semantic analysis. How could it? But of course the problem will reappear once you try to get a formal grasp on the pragmatics of believe ascriptions.
Attitudes in Modal Logic
Modal logics are often used for modelling propositional attitudes.

(18) Ralph believes that Ortcutt is a spy.

Simplified:

\[
\text{believe}(x, y) \quad \{w | \text{Ortcutt is a spy in } w\}
\]

the set of all worlds in which Ortcutt is a spy

How it really works:

(19) \(M, g, w \models B_a P(b)\)

\[\uparrow\]

(20) In all \(w'\) such that \(wR_a w'\): \(M, g, w' \models P(b)\)

\[\uparrow\]

(21) In all \(w'\) such that \(wR_a w'\): \(T_g(b) \in I(P, w')\)
Modal Logic Reminder
Recall: A model $M = \langle W, R, D, I \rangle$ is a Kripke model for a first-order modal logic consisting of a set of possible worlds $W$, an accessibility relation $R$, the domain $D$, and interpretation function $I$. 
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Propositional Attitudes and Referential Opacity - p. 12/23
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- $B_a$ is an operator like $\square$. 
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- $[i]$ and $\langle i \rangle$ are also often used as a notation for operators corresponding to $\Box$ and $\Diamond$ with accessibility relation $R_i$ in multi-modal systems. We’ll stick to $B_i$ for belief and $\Box$ for alethic modality.
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- $B_\alpha$ is an operator like $\Box$.

- So $M, g, w \models B_\alpha P(b)$ informally means something like $P(b)$ is true in all worlds that are compatible with what Ralph believes in $w$.

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- Limitations: We have a fixed set of agents represented as indices. By default, it’s not possible to quantify over agents, as this would require us to perform advanced set theoretic operations over various relations $R_i, R_j, \ldots$
A Problem With Doxastic Modalities

In normal modal logic, referential opacity cannot be encoded using ordinary constants. Ordinary constants are rigid with respect to any modality. Suppose \( M = \langle W, R, D, I \rangle \) such that \( I(c, w) = I(c, w') \) for any constant \( c \) and \( w, w' \in W \). Let \( a, b \) be constants that denote the same object, i.e. \( I(a, w) = I(b, w) \).

\[
\begin{align*}
(22) & \quad M, g, w \models B_P(a) \\
& \quad \upharpoonright \text{rule for belief operator} \\
(23) & \quad \text{in all } w' \text{ such that } wRw': M, g, w' \models P(a) \\
& \quad \upharpoonright \text{rules for predicates and constants} \\
(24) & \quad I(a, w') \in I(P, w') \\
& \quad \upharpoonright \text{by rigidity} \\
(25) & \quad I(a, w) \in I(P, w') \\
& \quad \upharpoonright \text{because } I(a, w) = I(b, w) \\
(26) & \quad I(b, w) \in I(P, w') \\
& \quad \upharpoonright \text{like above, but in reverse direction} \\
(27) & \quad M, g, w \models B_P(b)
\end{align*}
\]
A Possible Solution?

Co-referential constants are substitutable within the scope of doxastic modalities because they are rigid no matter which kind of modality is involved. Make them non-rigid in respect to doxastic modalities and the problem seems to be solved.

We can, it seems, still define rigidity for alethic modalities:

- Rigidity in respect of $\Box$: For any constant $c$, any $w_0$, and all $w_1, w_2$ such that $w_0 R_{\Box} w_1$ and $w_0 R_{\Box} w_2$: $I(c, w_1) = I(c, w_2)$.

- Non-rigidity in respect of $\mathcal{B}$: For any constant $c$, any $w_0$, and doxastic modality $i$, there can be two worlds $w_1, w_2$ such that $w_0 R_i w_1$ and $w_0 R_i w_2$ and $I(c, w_1) \neq I(c, w_2)$.

An example:

\[
\begin{align*}
  w_0 R_{\Box} w_0 & \quad I(a, w_0) = a \\
  w_0 R_{\Box} w_1 & \quad I(a, w_1) = a \\
  w_0 R_{\Box} w_2 & \quad I(a, w_2) = a \\
  w_0 R_{\Box} w_3 & \\
  w_0 R_a w_3 & \quad I(a, w_3) = a \\
  w_0 R_a w_4 & \quad I(a, w_4) = b
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- **Non-rigidity in respect of $\mathcal{B}$:** For any constant $c$, any $w_0$, and doxastic modality $i$, there can be two worlds $w_1, w_2$ such that $w_0 R_i w_1$ and $w_0 R_i w_2$ and $I(c, w_1) \neq I(c, w_2)$.

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 w_0 R \Box w_3 & I(a, w_3) = a \\
 w_0 R_i w_3 & I(a, w_4) = b \\
\end{array}
\]

- Why doesn’t this work?
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- Formally: $\neg \forall w_0, w_1 (w_0 R_B w_1 \rightarrow w_0 R_\square w_1)$.
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- Let’s call the set of worlds that are reachable by a doxastic accessibility relation $W_B$ and the set of worlds that are reachable by an alethic accessibility relation $W_{□}$. 
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- The proposed solution only works if $R\Box$ doesn’t grant access to all worlds that are compatible with any agent’s belief worlds.
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- But then, whatever statements this world makes true are impossibly true. ($\neg\Diamond \phi$ is true in any such world.)
- This is not very plausible.
- At least it’s not desirable for solving referential opacity, since it is unreasonable to claim that an agent believes something impossible when he just doesn’t know that two names are co-referential.
More Problems

Overgeneration

As a consequence of the following properties of normal modal logics more formulas are valid than might be desired.

Omniscience
If $A$ is provable, then it is also known or believed. So every logical tautology is automatically believed. The agent is logically omniscient. Necessitation rule: If $\vdash A$ then $\vdash \Box A$.

Distribution of $\land$ and $\rightarrow$
If an agent believes/knows that $A \land B$, he also believes/knows that $A$ and believes/knows that $B$. This is sometimes considered inplausible. In $K$ $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$ (K-Rule) and $\Box(p \land q) \rightarrow (\Box p \land \Box q)$ are valid, but not the disjunctive version.
Even More Problems

Undergeneration

As a result of the following properties of normal modal logics, less formulas are valid than might be desired.

Substitutability of Rigid Constants

In a standard treatment, constants are rigid and therefore substitutable with co-referential constants in any context. Making terms non-rigid in respect to one modality but rigid with respect to another is difficult, not standard, and not trivial.

Inner-world Substitutability

In a normal first-order modal logic, singular terms that are co-referential in one world are substitutable for each other in relation to this world. This has been considered problematic in connection with essential indexicals. If \( T_g(t_1, w) = T_g(t_2, w) \), then \( M, g, w \models A \iff A[t_1/t_2] \).
The approaches to deal with propositional attitudes can be classified according to several criterias.

- **Syntactic Approach**: quote a formula or linguistic entities themselves
  - **Direct Quotation**: use a quoting operator in the formal language itself: \( B(a, \lnot P(b)) \) “Quine-corners”. In combination with strong reflection principles, inconsistencies can arise (Montague’s theorem), but there are ways to deal with them. Lit. Carnap (1947), Davidson (1969), Wessel (1998), Bolander (2003)
  - **Indirect Quotation**: refer to linguistic material in another language, e.g. the natural language that is analysed, or syntactic material that is represented in parallel with the actual formulas. Lit. Token-dependency Semantics, Dahllöf (2002)

- **Non-syntactic Approach**: modify modal logics, use other logics
  - **Impossible Possible Worlds**: add impossible worlds and syntactic assignments. Lit. Hintikka (1967, 1979)
  - **Situation Semantics**: use fine-grained situations instead of possible worlds. Lit. Barwise & Perry (1983)
Kripke’s Puzzle
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Living in France, Pierre comes to believe that London is pretty. Later, when he is living in London, he comes to believe that London is ugly, but doesn’t realize that »Londres« and »London« denote the same city.

(28) Pierre: Londres est jolie
    ➟ Pierre believes that London is pretty.

(29) Pierre: London is ugly.
    ➟ Pierre believes that London is ugly.

Disquotation

The puzzle: We conclude that Pierre has contradictory beliefs, yet he seems to be a perfectly rational agent. Pierre hasn’t changed his beliefs. Does he believe that London is pretty or doesn’t he believe that London is pretty?
**Principle of Disquotation**

For an English sentence $p$:

“If a normal English speaker, on reflection, sincerely assents to ‘$p$’, then he believes that $p$.” (Kripke 1979, 112/113)
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- The principle assumes that the speaker reflects what he says. (He is not just babbling like an idiot or parrot.)
- Kripke’s puzzle assumes two disquotational principles, one for French and one for English.
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For languages:

“If a sentence of one language expresses a truth in that language, then any translation of it into any other language also expresses a truth (in that other language).” (Kripke 1979, 112/113)
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- Question: Does this principle also apply to believe sentences with referentially opaque reading?
Some Questions About Belief Ascriptions

- What are the criteria for...

- In what way does a sentence express an attitude of the speaker?
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