Introduction to the Philosophy of Language

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Rigid Designation

To understand a proposition means to know what it is the case, if it is true. (One can therefore understand it without knowing whether it is true or not.) One understands it if one understands its constituent parts.

(Wittgenstein, Tractatus Logico-Philosophicus, 4.024)

Literature

- Kripke (1972): Naming and Necessity. (excerpts)
- Chapter 3 and 4 of Lycan (2000)

Background: Kripke

Saul Kripke (*1940): Background

- Kripke invents the first semantics for modal logic (1963) and proves completeness of ML (1959).
- Possible world semantics is still the standard semantics for ML, and all so-called normal modal logics are based on system K (K in honor of Kripke but just pronounced "kreep").
- His controversial book Naming and Necessity (1972) is the foundation of the New Theory of Reference.
- Kripke also works on the interpretation of the late Wittgenstein, Wittgenstein on Rules and Private Language (1982) is a classical, “must-read” text on Wittgenstein (or, as some less benevolent readers claim, on Kripkenstein).
- Kripke published a vast number of articles and nearly all of them were highly influential. Examples: Identity and Necessity (1971), Outline of a Theory of Truth (1975), A Puzzle about Belief (1979), Speaker’s Reference and Semantic Reference (1979)
- There is a rather silly controversy initiated by Quentin Smith whether some of Kripke’s main theses were already proposed by Ruth Barcan Marcus. (Kripke’s work is based on Marcus’ work, so you should read her publications as well.)
**Syntax of First-Order Modal Logic**

All formulas of first-order predicate logic are formulas of first-order modal logic, plus the box and the diamond operator:

<table>
<thead>
<tr>
<th>Formula</th>
<th>→</th>
<th>Pred(Terms)</th>
<th>(Formula ∧ Formula)</th>
<th>→</th>
<th>Formula</th>
<th>∃ Var Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terms</td>
<td>→</td>
<td>Const</td>
<td>Var</td>
<td>Terms, Terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Const</td>
<td>→</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>Const'</td>
<td></td>
</tr>
<tr>
<td>Var</td>
<td>→</td>
<td>x</td>
<td>y</td>
<td>z</td>
<td>Var'</td>
<td></td>
</tr>
<tr>
<td>Pred</td>
<td>→</td>
<td>give</td>
<td>laugh</td>
<td>slap</td>
<td>love</td>
<td>hate</td>
</tr>
</tbody>
</table>

So we can write things like:

1. \(\square \text{slap}(a, b)\)
   
   which could have a reading as in

2. It is necessary that Mary slaps Peter.
   
   but it could also mean

3. Peter believes that the Morning Star is identical to the Evening Star.

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**Semantics of ML**

**Model of ML:** A Kripke model for first-order modal logic is an ordered 4-tuple \(M = (W, R, D, I)\) where

- \(D\) is a non-empty domain of objects
- \(I\) is an interpretation function for constants and predicates such that
  
  - \(I(c, w) \in D\), for each constant \(c\)
  
  - \(I(P, w) \subseteq D^n\), i.e. \(D \times \cdots \times D\), for each predicate \(P\) of arity \(n\)
- \(W\) is a non-empty set of possible worlds, states, or situations.
- \(R\) is a binary relation on \(W\), i.e. it is a subset of \(W \times W\).

**Kripke Frame:** Note: A structure \(F = (W, R)\) is called a Kripke frame, and modal logics based on it are called normal modal logics.

**Assignment Function.** Like in PL1, an assignment \(g\) is a function from variables to elements in \(D\).

**Term Interpretation.** Let \(T_g(x, w)\) be a function from variables or constants and worlds to elements in \(D\) with respect to an assignment \(g\), such that...

- \(T_g(t, w) = g(t)\) if \(t\) is a variable, and
- \(T_g(a, w) = I(t, w)\) if \(a\) is a constant.

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**Revised Syntax**

Here is a revised syntax that looks a bit more abstract:

<table>
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<td>Var'</td>
<td></td>
</tr>
<tr>
<td>Pred</td>
<td>→</td>
<td>P</td>
<td>F</td>
<td>G</td>
<td>R</td>
<td>Pred'</td>
</tr>
</tbody>
</table>

- The modal logic part is in the two operators \(\Box\) and \(\Diamond\).
- The rest is first-order predicate logic.
- By convention, let’s write \(x_1, x_2, P_1, \ldots\) for \(x', x'', P''', \ldots\) respectively.
- Like with first-order predicate logic, let’s stipulate that every predicate has a fixed arity.

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**Truth in a Model**

**Truth in a Model.** Truth in a model \(M\) with respect to an assignment \(g\) is defined by the following rules.

1. \(M, g, w \models P(t_1, \ldots, t_n)\) if \(\langle t_1, w \rangle, \ldots, \langle t_n, w \rangle \in I(P, w)\)
2. \(M, g, w \models A \land B\) if \(M, g, w \models A\) and \(M, g, w \models B\)
3. \(M, g, w \models \neg A\) if it is not the case that \(M, g, w \models A\)
4. \(M, g, w \models \exists v A\) if there is a \(v\)-variant \(h\) of \(g\) such that \(M, h, w \models A\)
5. \(M, g, w \models \Box A\) if in all worlds \(w'\) such that \(wRw'\) it is the case that \(M, g, w' \models A\)
6. \(M, g, w \models \Diamond A\) if there is a world \(w'\) such that \(wRw'\) and \(M, g, w' \models A\)
Remarks

- You can express the same in first-order predicate logic as in first-order modal logic. (In some sense, first-order modal logic is just a syntactic variant of first-order predicate logic.)
- The modal operators □ and ◇ are hidden universal and existential quantifiers.
- But the quantification is guarded by the accessibility relation.
  \[\phi \text{ iff } \forall u'[\mathcal{R}(u, u') \rightarrow \psi(u)']\]
- There are two important choices to make:
  * You have to choose whether constants are interpreted in respect to a world or not (non-rigid versus rigid ‘constants’).
  * You have to choose whether the first-order quantifiers \(\exists\) and \(\forall\) are interpreted in respect to a world or not (actualist versus possibilist quantification).
- Depending on which properties the accessibility relation \(\mathcal{R}\) has, you get different modal logics \(\mathcal{K}\), \(\mathcal{K}t\), \(\mathcal{Kd}\), \(\mathcal{Kd}45\), \(\mathcal{S}4\), \(\mathcal{S}5\), \ldots
- All of them are called normal, because they are based on Kripke frames.
- \([A \rightarrow B] \rightarrow (\Box A \rightarrow \Box B)\) holds in \(\mathcal{K}\), thus in all normal modal logics.
- If \(F \models A\), i.e. \(A\) is valid in a Kripke frame \(F\) (=\(A\) is true in all models based on \(F\)), then \(F \models \Box A\) as well (necessitation rule).

Applications

Modal logic has been used or abused for a variety of tasks in philosophy:

Aesthetic Modalities
- It is necessary that 2 is equal to 2.
  \[\Box \text{equal}(2, 2)\]
- It is possible that Peter is rich.
  \[\Diamond \text{rich}(Peter)\]

Temporal Modalities
- Peter will be rich.
  \[\text{rich}(Peter)\]
- Peter will always be rich.
  \[\Box \text{rich}(Peter)\]

Doxastic Modalities
- Mary believes that Peter is rich.
  \[KD45 \vdash \Box \text{rich}(Peter)\]
- Mary knows that Peter is rich.
  \[S5 \vdash \Box \text{rich}(Peter)\]

Deontic Modalities
- If it is obligatory that someone pays the bill \(\Box \exists x[\text{pay}(x, \#\text{bill}(y))]\) then it is permitted that someone pays the bill \(\Diamond \exists x[\text{pay}(x, \#\text{bill}(y))]\)

Rigid Designation

Kripke’s main thesis: Proper names are rigid designators.

A proper name \(t\) is a rigid designator if \(t\) denotes the same individual no matter in the scope of which expression it occurs.

Example:

1. Aristotle might not have been the teacher of Alexander.
2. The teacher of Alexander was interpreted relative to the contrafactual situation, but "the teacher of Alexander" is interpreted contrafactually.
3. Compare with:
4. Aristotle could not have been called "Aristotle".
5. The teacher of Alexander could not have been the teacher of Alexander.

Interpretation: Kripke claims that proper names are rigid designators, whereas definite descriptions are interpreted with respect to the contrafactual situation that is described.
The New Theory of Reference

- **Reference** is fixed by an initial act of baptism (e.g. by using ostension or a description).
- The name is passed from "link to link", at each occasion of use, as it is used in the community.
- The hearer must **intend** to use the name with the same reference as the speaker he heard it from.

Exkurs: Scope

The definition of rigid designators was my own, and I've just realized that there's a problem with it. Sorry! It's quite 'educational' to see what's the problem.

**Scope in Formal Languages**

Scope in formal languages is often indicated by implicit or explicit parentheses and/or by variable binding:

\[
\exists x \forall y \phi(a, x) \quad \exists (x, \phi((\alpha, x)))
\]

This doesn't have to affect scope, because the interpretations can ensure that the formulas have the same semantics—because they are intended to be PL1 formulas.

**Scope in Natural Language**

The syntactic structure of natural languages is extracted from empirical observations. In generative grammar, it is based on the notion of **constituency**. The notion of scope in a natural language only makes sense when a concrete syntax and a concrete semantical representation is considered. In other words, the notion is at the core of the syntax-semantics interface.

Exkurs: Syntax and Scope

The syntactic structure in formal languages is arbitrary:

- **Infix Notation**: \[\exists x[\neg P(x) \land Q(x)] \lor F(x)]\]
- **Polish Notation**: \[Sx.ACNpQxFx\]
- **Reverse Polish Notation**: \[xPnQxCxFAvS\]

Exkurs: Scope in Formal Languages
**Scope in Natural Languages**

Neither the syntax nor the notion of scope is arbitrary in natural languages. Conversely, the syntax-semantics interface is highly theory-dependent. For example, scope might be defined by the notion of c-command:

```
    it is possible that
     C
     /
   CP
     /
 NP
  Peter
      /
 VP
    over
 NP
  Mary
```

The blue part of the tree lies within the c-command domain of "it is possible that", thus it is in its scope. C-command is a syntactic notion, structurally defined on trees. It might be regarded as determining scope, but scope can also be regarded as a purely semantic notion.

**Dominance and C-Command**

Just as an example, here's one possible definition of dominance (my own formulation):

\[ \alpha \text{ dominates } \beta \text{ iff } \alpha \text{ is nearer to the root of the tree than } \beta \text{ and there's a path from } \alpha \text{ to } \beta. \]

Here's one possible definition of c-command, after Chomsky (1986):

\[ \alpha \text{ c-commands } \beta \text{ iff } \alpha \text{ does not dominate } \beta \text{ and every node } \gamma \text{ that dominates } \alpha \text{ also dominates } \beta. \]

**Syntax–Semantics Interface**

Sometimes, syntactic and semantic notions of scope do not match up:

```
    S
    /
  DP
  3 somebody 3 everybody
      /
    S
      /
  DP
  3 everybody 3 someone
      /
    S
      /
  DP
```

"Everybody loves someone"

1. \( \exists y \forall x [\text{love}(x, y)] \)
2. \( \forall x \exists y [\text{love}(x, y)] \)

Both readings require quantifier raising. C-command alone doesn't help us in explaining the scope of "someone".

**Summary**

So what's the moral of this little intermezzo?

- There's no 1 to 1 connection between syntax and semantics. The same syntactic structure is sometimes considered ambiguous between several semantic readings.
- Whether some natural language expression like a proper name lies within the scope of another expression can depend on:
  - ...concrete ways of how a semantical respecification is obtained from some concrete syntax
  - ...a certain semantic reading that is stipulated for the meaning of an expression in dependence on the meaning of another expression
- The two notions should coincide, but sometimes they might not.
- Question: Is a proper name according to Kripke, in the scope or outside the scope of "it is necessary that"?
  - Answer 1: inside (syntactic notion of scope)
  - Answer 2: outside (semantic notion of scope)
- Perhaps it's better not to use the notion of scope but rather speak of semantic dependence.
**Rigid Designation (continued)**

**Rigid Designation versus Senses**

Direction of Kripke’s Argumentation:

- Kripke argues against Frege’s view that proper names have a sense and the Russellian view that proper names are definite descriptions in disguise.

- Argument Structure
  - The Fregian sense or a definite description of a proper name would have to be rigid/independent of modal expressions.
  - There doesn’t seem to be any rigid property, complex description, or ‘sense’ in any such case.

- Kripke mostly argues with natural language examples.
  - He tries not to presume possible worlds semantics as a framework of analysis. (It’s not clear whether he succeed in this, though.)

**Caveats**

- Modal operators can easily be mixed up with natural language expressions that are analysed as modal operators, definite descriptions with iota-terms, proper names with individual constants, and so on. But formal properties of expressions in a formal language under some interpretation only tell us something about corresponding natural language expressions, if the corresponding analysis is correct, appropriate, descriptively adequate, etc.

- The term rigid designation is often used both for expressions in formal languages and in natural language, but then you need to be aware that you use the term equivocally.

- Kripke’s metaphysical claims about necessity have to be separated from his claims about how we interpret expressions in natural languages.

- Different kind of ‘modalities’ might need to be treated separately.

- Modality is a technical, artificial, philosophical concept. The notion is based on normal modal logic, not on how people usually interpret expressions like «to believe», «to ought to», «it is necessary that».

- Perhaps some kind of modalities don’t make any sense. (See Quine’s critique on modal logic.)

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**Rigid Designation—Revision**

Kripke’s Thesis (revised): Proper names are rigid designators.

- A proper name is a rigid designator if it is semantically independent from certain modal expressions like «it is necessary that» or «it is possible that».

- Unfortunately, that doesn’t seem to be what Kripke really says.

- Instead, he argues directly against Russellian and Fregian analyses of proper names.

- The definition is pretty vague anyway.
Formal Definition of Rigid Designation

In a possible worlds framework, rigid designation can be defined precisely without problems:

A term \( \ell \) is a rigid designator (with respect to arbitrary assignment \( \gamma \) and some model \( M = (W; R, D, I) \)) iff for all worlds \( w, w' \in W: T_M(\ell, w) = T_M(\ell, w') \).

- This just says that \( \ell \) denotes the same object in all worlds.
- So proper names could be terms that are rigid designators.
- However, this definition doesn’t make any distinction with respect to different kind of modalities, because it is quantified over all possible worlds, no matter whether they are accessible or not.
- So far, we have only one kind of modality in our logic.
- But ML can easily be extended to have many different kind of modalities \( \Box \), each with its own accessibility relation \( R_\Box \).
- Then the definition has to be adjusted to take into account the respective \( R_\Box \).

Essential Properties

Essential properties are one reply to Kripke, but perhaps not a very good one.

For each proper name, there’s a description that picks out the same object under any kind of counterfactual circumstances.

In other words, a description is stipulated that is unique and itself rigid.

(7) Aristotle might not have been the teacher of Alexander.

(8) \( \Box z(\exists x \neg Gx) \neq \gamma y(Fy) \)

where for all \( w, w' \in W: T_M(\exists x(\Box(\neg Gx)), w) = T_M(\exists x(\Box(\neg Gx)), w') \)

So \( \neg G \) is a necessary or essential property of Aristotle. What would that be? Is that evil metaphysics or great ontology?

Objections to Kripke

Wide Scope Theory

The wide scope theory is one possible reply to Kripke from a Russelian perspective. Russell's thesis for proper names, revised for modal expressions:

Proper names are definite descriptions in disguise. They take wide scope over modal operators.

Consider the following examples (some parentheses omitted):

(9) Aristotle might not have been the teacher of Alexander.

(10) \( \exists x(Fx)[\Box y(Fy)(x \neq y)] \)

(11) \( \exists x[Fx][\Box x \neq \gamma y(Fy)] \)

(12) \( \exists x[\Box x \neq \gamma y(Fy) \land \Box x \neq \Box z(Fz)] \)

In all of these examples, the first description has wide scope over \( \Box \). This is also called a de re reading. This ensures that the first description is interpreted independently of \( \Box \). (Note: The same can also be done with \( \lambda \)-abstraction.)
**Actuality Operator**

An actuality operator in a definite description is another way to reply to Kripke. So we say something like “the unique person that is actually the so-and-so, under some counterfactual circumstances is such-and-such”.

Proper names are definite descriptions in disguise. These descriptions always pick out an object in the actual world.

Two basic ways to get an actuality operator: use hybrid logic with \( \downarrow \) and \( \& \), or add a designated world \( w_0 \) to the model that is the actual world:

13. \( M, g, w \models Actually.A \iff M, g, w_0 \models A \)
where \( M = \langle W, R, D, I, w_0 \rangle \) and \( w_0 \in W \)
14. Aristotle might not have been the teacher of Alexander.
15. \( \square \forall x (ActuallyFx) \models \forall y (Fy) \)

**Temporal Modalities**

So far, the operators have always been interpreted as «it is necessary that» and «it is possible that». That is *alethic modalities*. But what about other modalities?

**Proper Names and Temporal Modalities**

16. Aristotle was the teacher of Alexander.
17. Aristotle is the teacher of Alexander.

«Aristotle» doesn’t refer to different individuals depending on the tense of the main clause, so it is rigid in respect to temporal modalities. Note that questions of possibility versus actualism arise in temporal interpretations. Formally, the past tense in «was» can be analysed in a modal logic as a temporal operator \( F \) (for past), which behaves like \( \square \) in \( K \) with a transitive accessibility relation. But we need another operator \( F \) for the future as well. The minimal system of tense logic is called \( K_{t} \) ("K sub t"). It is based on the work of A.N. Prior. Literature: Goldblatt 1992, *Logics of Time and Computation*.

**Doxastic Modalities**

**Proper Names and Doxastic Modalities**

This is related to *propositional attitudes*, and the problem of *referential opacity* in belief ascriptions. (topic of next session)