Rigid Designation

“To understand a proposition means to know what is the case, if it is true. (One can therefore understand it without knowing whether it is true or not.) One understands it if one understands its constituent parts.”

(Wittgenstein, *Tractatus Logico-Philosophicus*, 4.024)

**Literature**

- **Kripke (1972): Naming and Necessity. (excerpts)**
- **Searle (1958): Proper Names.**
- **Chapter 3 and 4 of Lycan (2000)**
First-Order Modal Logic
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- There is a rather silly controversy initiated by Quentin Smith whether some of Kripke’s main theses were already proposed by Ruth Barcan Marcus. (Kripke’s work is based on Marcus’ work, so you should read her publications as well.)
**Syntax of First-Order Modal Logic**

All formulas of first-order predicate logic are formulas of first-order modal logic, plus the box and the diamond operator:

<table>
<thead>
<tr>
<th>Formula</th>
<th>$\rightarrow$</th>
<th>Pred(Terms)</th>
<th>(Formula $\land$ Formula)</th>
<th>$\neg$ Formula</th>
<th>$\exists$ Var Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terms</td>
<td>$\rightarrow$</td>
<td>Const</td>
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<td></td>
</tr>
<tr>
<td>Const</td>
<td>$\rightarrow$</td>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
<td>Const'</td>
</tr>
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<td>Var</td>
<td>$\rightarrow$</td>
<td>$x$</td>
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</tr>
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<td>Pred</td>
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\[
\begin{align*}
\text{Formula} & \rightarrow \text{Pred(Terms)} \mid (\text{Formula} \land \text{Formula}) \mid \neg \text{Formula} \mid \exists \text{Var Formula} \\
\text{Formula} & \rightarrow \square \text{Formula} \mid \Diamond \text{Formula} \\
\text{Terms} & \rightarrow \text{Const} \mid \text{Var} \mid \text{Terms, Terms} \\
\text{Const} & \rightarrow a \mid b \mid c \mid \text{Const}' \\
\text{Var} & \rightarrow x \mid y \mid z \mid \text{Var'} \\
\text{Pred} & \rightarrow \text{give} \mid \text{laugh} \mid \text{slap} \mid \text{love} \mid \text{hate} \mid \text{Book} \mid \ldots
\end{align*}
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\end{align*}$$

So we can write things like:

(1) $$\Box \text{slap}(a, b)$$

which could have a reading as in

(2) It is necessary that Mary slaps Peter.

but it could also mean

(3) Peter believes that the Morning Star is identical to the Evening Star.
Here is a revised syntax that looks a bit more abstract:

\[\begin{align*}
\text{Formula} & \rightarrow \mathbf{Pred}(\text{Terms}) \mid (\text{Formula} \land \text{Formula}) \mid \neg \text{Formula} \mid \exists \text{Var} \text{Formula} \\
\text{Formula} & \rightarrow \Box \text{Formula} \mid \Diamond \text{Formula} \\
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\text{Const} & \rightarrow a \mid b \mid c \mid \text{Const}' \\
\text{Var} & \rightarrow x \mid y \mid z \mid \text{Var}' \\
\text{Pred} & \rightarrow P \mid F \mid G \mid R \mid \text{Pred}'
\end{align*}\]

- The modal logic part is in the two operators $\Box$ and $\Diamond$.
- The rest is first-order predicate logic.
- By convention, let's write $x_1, x_2, P_3, \ldots$ for $x', x'', P''', \ldots$ respectively.
- Like with first-order predicate logic, let's stipulate that every predicate has a fixed arity.
Model of ML  A Kripke model for first-order modal logic is an ordered 4-tupel 
\[ M = \langle W, R, D, I \rangle \] where

- \( D \) is a non-empty domain of objects
- \( I \) is an interpretation function for constants and predicates such that
  - \( I(c, w) \in D \), for each constant \( c \)
  - \( I(P, w) \subseteq D^n \), i.e. \( D \times \cdots \times D \), for each predicate \( P \) of arity \( n \)
- \( W \) is a non-empty set of possible worlds, states, or situations.
- \( R \) is a binary relation on \( W \), i.e. it is a subset of \( W \times W \).

Kripke Frame  Note: A structure \( F = \langle W, R \rangle \) is called a Kripke frame, and modal logics based on it are called normal modal logics.

Assignment Function.  Like in PL1, an assignment \( g \) is a function from variables to elements in \( D \).

Term Interpretation.  Let \( T_g(x, w) \) be a function from variables or constants and worlds to elements in \( D \) with respect to an assignment \( g \), such that . . .

- \( T_g(t, w) = g(t) \) if \( t \) is a variable, and
- \( T_g(t, w) = I(t, w) \) if \( a \) is a constant.
**Truth in a Model.** Truth in a model $M$ with respect to an assignment $g$ is defined by the following rules.

1. $M, g, w \models P(t_1, \ldots, t_n)$ iff $\langle T_g(t_1, w), \ldots, T_g(t_n, w) \rangle \in I(P, w)$
2. $M, g, w \models A \land B$ iff $M, g, w \models A$ and $M, g, w \models B$
3. $M, g, w \models \neg A$ iff it is not the case that $M, g, w \models A$
4. $M, g, w \models \exists v A$ iff there is a $v$-variant $h$ of $g$ such that $M, h, w \models A$
5. $M, g, w \models \Box A$ iff in all worlds $w'$ such that $wRw'$ it is the case that $M, g, w' \models A$
6. $M, g, w \models \Diamond A$ iff there is a world $w'$ such that $wRw'$ and $M, g, w' \models A$
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$\Box (A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$ holds in $K$, thus in all normal modal logics.

If $F \vDash A$, i.e. $A$ is valid in a Kripke frame $F$ (= $A$ is true in all models based on $F$), then $F \vDash \Box A$ as well (**necessitation** rule).
Modal logic has been used or abused for a variety of tasks in philosophy:

### Alethic Modalities

- It is necessary that 2 is equal to 2. \( \square \text{equal}(2,2) \)
- It is possible that Peter is rich. \( \Diamond \text{rich}(Peter) \)

### Temporal Modalities

- Peter will be rich. \( \Diamond \text{rich}(Peter) \)
- Peter will always be rich. \( \square \text{rich}(Peter) \)

### Doxastic Modalities

- Mary believes that Peter is rich. \( KD45 \models \square \text{rich}(Peter) \)
- Mary knows that Peter is rich. \( S5 \models \square \text{rich}(Peter) \)

### Deontic Modalities

- If it is obligatory that someone pays the bill then it is permitted that someone pays the bill

\[
\begin{align*}
\square & \exists x \left[ \text{pay}(x, \, \text{bill}(y)) \right] \\
& \rightarrow \Diamond \exists x \left[ \text{pay}(x, \, \text{bill}(y)) \right]
\end{align*}
\]
Rigid Designation
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Kripke’s main thesis: Proper names are **rigid designators**.

A proper name $t$ is a rigid designator iff $t$ denotes the same individual no matter in the scope of which expression it occurs.

Example:

(4) Aristotle might not have been the teacher of Alexander.

Interpretation: »Aristotle« is not interpreted relative to the contrafactual situation, but »the teacher of Alexander« is interpreted contrafactually. Compare with:

(5) Aristotle could not have been called »Aristotle«.
(6) The teacher of Alexander could not have been the teacher of Alexander.

Interpretation: Kripke claims that proper names are rigid designators, whereas definite descriptions are interpreted with respect to the contrafactual situation that is described.
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Exkurs: Scope
The definition of rigid designators was my own, and I’ve just realized that there’s a problem with it. Sorry! It’s quite ‘educational’ to see what’s the problem.

**Scope in Formal Languages**

Scope in formal languages is often indicated by implicit or explicit parentheses and/or by variable binding:

\[
\exists x \Box \Diamond P(a, x) \quad \exists (x, \Box (\Diamond (a, x)))
\]
The syntactic structure in formal languages is arbitrary:

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**Scope in Natural Language**

The syntactic structure of natural languages is extracted from empirical observations. In generative grammar, it is based on the notion of **constituency**. The notion of scope in a natural language only makes sense when a concrete syntax and a concrete semantical representation is considered. In other words, the notion is at the core of the **syntax–semantics interface**.
Neither the syntax, nor the notion of scope is arbitrary in natural languages. Conversely, the syntax-semantics interface is highly theory-dependent. For example, scope might be defined by the notion of **c-command**:
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The **blue** part of the tree lies within the c-command domain of »it is possible that«, thus it is in its scope. C-command is a syntactic notion, structurally defined on trees. It might be regarded as determining scope, but scope can also be regarded as a purely semantic notion.
Dominance and C-Command

Just as an example, here's one possible definition of dominance (my own formulation):

\[ \alpha \text{ dominates } \beta \text{ iff } \alpha \text{ is nearer to the root of the tree than } \beta \text{ and there's a path from } \alpha \text{ to } \beta. \]

Here's one possible definition of c-command, after Chomsky (1986):

\[ \alpha \text{ c-commands } \beta \text{ iff } \alpha \text{ does not dominate } \beta \text{ and every node } \gamma \text{ that dominates } \alpha \text{ also dominates } \beta. \]
Syntax–Semantics Interface

Sometimes, syntactic and semantic notions of scope do not match up:
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```
S'  
  DP  ① someone  ② everybody  
     S  
       DP  ① everybody  ② someone  
         VP  
            V loves  
               DP t

"Everybody loves someone"
① \( \exists y \forall x [love(x, y)] \) or ② \( \forall x \exists y [love(x, y)] \)
Both readings require quantifier raising. C-command alone doesn’t help us in explaining the scope of »someone«.
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  - Answer 1: inside (syntactic notion of scope)
  - Answer 2: outside (semantic notion of scope)
- Perhaps it’s better not to use the notion of scope but rather speak of semantic dependence.
Rigid Designation (continued)
Kripke’s Thesis (revised): Proper names are rigid designators.

A proper name is a rigid designator iff it is semantically independent from certain modal expressions like »it is necessary that« or »it is possible that«.
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- Instead, he argues directly against Russellian and Fregean analyses of proper names.
Rigid Designation—Revision

Kripke’s Thesis (revised): Proper names are rigid designators.

A proper name is a rigid designator iff it is semantically independent from certain modal expressions like »it is necessary that« or »it is possible that«.

- Unfortunately, that doesn’t seem to be what Kripke really says.
- Instead, he argues directly against Russellian and Fregean analyses of proper names.
- The definition is pretty vague anyway.
Direction of Kripke’s Argumentation:
**Rigid Designation versus Senses**

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- Kripke argues against Frege’s view that proper names have a sense and the Russellian view that proper names are definite descriptions in disguise.
Rigid Designation versus Senses

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  - There doesn’t seem to be any rigid property, complex description, or ‘sense’ in any such case.
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- Kripke mostly argues with natural language examples.

- He tries not to presume possible worlds semantics as a framework of analysis. (It’s not clear whether he succeed in this, though.)
Caveats
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- Modal operators can easily be mixed up with natural language expressions that are analysed as modal operators, definite descriptions with iota-terms, proper names with individual constants, and so on. But formal properties of expressions in a formal language under some interpretation only tell us something about corresponding natural language expressions, if the corresponding analysis is correct, appropriate, descriptively adequate, etc.
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- Modality is a technical, artificial, philosophical concept. The notion is based on normal modal logic, not on how people usually interpret expressions like »to believe«, »to ought to«, »it is necessary that«.

- Perhaps some kind of modalities don’t make any sense. (See Quine’s critique on modal logic.)
In a possible worlds framework, rigid designation can be defined precisely without problems:

A term $t$ is a rigid designator (with respect to arbitrary assignment $g$ and some model $M = \langle W, R, D, I \rangle$) iff for all worlds $w, w' \in W$: $T_g(t, w) = T_g(t, w')$. 
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- So far, we have only one kind of modality in our logic.
- But ML can easily be extended to have many different kind of modalities \( \Box_i \), each with its own accessibility relation \( R_i \).
- Then the definition has to be adjusted to take into account the respective \( R_i \).
Objections to Kripke
**Essential Properties**

Essential properties are one reply to Kripke, but perhaps not a very good one.

For each proper name, there's a description that picks out the same object under any kind of counterfactual circumstances.

In other words, a description is stipulated that is unique and itself rigid.

(7) Aristotle might not have been the teacher of Alexander.

(8) $\Diamond x(Gx) \neq \Diamond y(Fy)$

where for all $w, w' \in W : T_g(\Diamond x(Gx), w) = T_g(\Diamond x(Gx), w')$

So $G$ is a necessary or essential property of Aristotle. What would that be? Is that evil metaphysics or great ontology?
Wide Scope Theory

The wide scope theory is one possible reply to Kripke from a Russellian perspective. Russell's thesis for proper names, revised for modal expressions:

Proper names are definite descriptions in disguise. They take wide scope over modal operators.

Consider the following examples (some parentheses omitted):

(9) Aristotle might not have been the teacher of Alexander.

(10) $\forall x[Fx][\Diamond \forall y(Fy)(x \neq y)]$

(11) $\forall x[Fx][\Diamond x \neq \forall y(Fy)]$

(12) $\exists x[x = \forall y(Fy) \land \Diamond x \neq \forall z(Fz)]$

In all of these examples, the first description has wide scope over $\Diamond$. This is also called a de re reading. This ensures that the first description is interpreted independently of $\Diamond$. (Note: The same can also be done with $\lambda$-abstraction.)
An **actuality operator** in a definite description is another way to reply to Kripke. So we say something like “the unique person that is actually the so-and-so, under some counterfactual circumstances is such-and-such”.

Proper names are definite descriptions in disguise. These descriptions always pick out an object in the actual world.

Two basic ways to get an actuality operator: use **hybrid logic** with ↓ and @, or add a **designated world** $w_0$ to the model that is the actual world:

(13) $M, g, w \models \text{Actually}A$ iff $M, g, w_0 \models A$

where $M = \langle W, R, D, I, w_0 \rangle$ and $w_0 \in W$

(14) Aristotle might not have been the teacher of Alexander.

(15) $\Diamond x(\text{Actually}F x) \neq \forall y(F y)$
Temporal Modalities

So far, the operators have always been interpreted as »it is necessary that« and »it is possible that«. That is **alethic modalities**. But what about other modalities?

**Proper Names and Temporal Modalities**

(16) Aristotle was the teacher of Alexander.

(17) Aristotle is the teacher of Alexander.

»Aristotle« doesn’t refer to different individuals depending on the tense of the main clause, so it is rigid in respect to temporal modalities. Note that questions of possibilism versus actualism arise in temporal interpretations.

Formally, The past tense in »was« can be analysed in a modal logic as a temporal operator $P$ (for past), which behaves like $\Diamond$ in $K$ with a transitive accessibility relation. But we need another operator $F$ for the future as well. The minimal system of tense logic is called $K_t$ ("K sub t"). It is based on the work of A.N. Prior. Literature: Goldblatt 1992, *Logics of Time and Computation*. 
Doxastic Modalities

Proper Names and Doxastic Modalities

This is related to **propositional attitudes**, and the problem of **referential opacity** in **belief ascriptions**. (topic of next session)