ARGUMENT STRENGTH FOR BIPOLAR ARGUMENTATION GRAPHS

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Values in Argumentative Discourse (PTDC/MHC-FIL/0521/2014)

Value Seminar, June 2017
Overview

Some Terminology

Basic Argument Structure

Combinative Argument Strength

Problems
Acceptability-based Argument Strength

• Selinger (2014): Argument strength is based on an intuitive notion of acceptability.
• This is in contrast to probabilistic argument strength, which spells out argument strength in terms of the probability that the conclusion is true.
• Acceptability can be understood in two different ways:
  • a purely psychological notion $\leadsto$ perceived argument strength
  • an ideally rational notion $\leadsto$ argument strength

$\Rightarrow$ I investigate ideally rational argument strength like Selinger (2014). Note that a psychological account is entirely empirical, so you cannot make any informed conjectures about it without research in social psychology ($\sim$ rhetorics).
Positional vs. Intuitive Argument Strength

- Positional Argument Strength: The strength of an argument depends on its position in an argument graph.
  - AAFs (Dung 1995): unipolar - only attackers; not graded - arguments are either IN or OUT.
  - BAFs (Cayrol 2005, 2009): bipolar - attack and support; graded argument strength.

- BAF argument strength is positional in that the final strength of an argument hinges on its attackers and supporters.
- BAF argument strength is also intuitive and not only positional in that the positional calculation hinges on an initial assessment of individual arguments.

☞ I follow Cayrol & Lagasquie-Schiex (2009), Selinger (2014) and others in that (problematic) assumption: There is an initial assessment of argument strength, which is then modified in dependence of its attackers and supporters in a separate step.
For convergent arguments:

- **Ampliative Argument Strength**: The more pro (con) arguments for a standpoint, the higher (lower) the strength of that standpoint.
- **Nonampliative Argument Strength**: not ampliative.
- **Exclusionary Argument Strength**: The number of pro (con) arguments for a standpoint does not influence the strength of that standpoint.

(AP) Ampliativity: “If the quantity of the supports (resp. attacks) increases then the quality of the support (resp. attack) increases.” (Cayrol 2005, 78)

(EN) Neutrality: The number of pro (contra) arguments in a bundle on a does not influence the argument strength of a.

⚠️ I opt for EN, but also lay out ampliative principles.
My Own Proposal

**Exclusionary Account**

The account in this article is bipolar, graded, acceptability-based, and exclusionary, and also does justice to the distinction between linked and convergent arguments.
**Bipolar Argumentation Graphs**

- **Basic arguments**: a single link from premise to conclusion + polarity
- **Convergent arguments**: multiple basic arguments of the same polarity (considered as a unit)
- **Linked arguments**: multiple premises of given initial strength + link to conclusion + polarity + threshold for number of premises
- **Conductive arguments**: multiple premises of given initial strength and varying polarity + link to conclusion + polarity of link
- **Link attackers and link supporters**: any of the above structures can also attack a link instead of a premise or conclusion
- **Bipolar Argumentation Graph**: A finite, directed, acyclic and connected graph consisting of any of the above subgraphs.

⚠️ In contrast to BAFs and AAFs BAGs are *acyclic* and (without loss of generality) *connected*, but they need not form a tree.
**Convergent Argument Structure**

*Figure:* Structure of a convergent argument with initial strengths $x_1, \ldots, x_n$ and polarity $p$. 

$$
\begin{array}{c}
\text{a}_1 \\
p \\
x_1 \\
\downarrow \\
b
\end{array} \quad \begin{array}{c}
\text{a}_2 \\
p \\
x_2 \\
\downarrow \\
b
\end{array} \quad \begin{array}{c}
\text{a}_3 \\
p \\
x_3 \\
\downarrow \\
b
\end{array} \quad \cdots \quad \begin{array}{c}
\text{a}_n \\
p \\
x_n \\
\downarrow \\
b
\end{array}
$$
**Linked Argument Structure**

![Diagram of a linked argument structure with initial strengths $x_1, \ldots, x_n$ and polarity $p$.]

*Figure:* Structure of a linked argument with initial strengths $x_1, \ldots, x_n$ and polarity $p$. 
**Figure:** Structure of a conductive argument with initial strengths/values $x_1, \ldots, x_n$, individual polarities $p_1, \ldots, p_n$ and overall polarity $p$. 
**Figure:** An argument for or against the inference link of another argument with strength $x$ and polarity $q$; the links and premises $A$ and $C$ can be of any type.
Convergent Arguments

(EP) Exclusionary Principle: The combined strength of a pro (contra) convergent argument is the maximum of the strengths of its basic arguments.

(ACU) Cumulative Principle: The strength of convergent arguments consisting of $n > 1$ independent basic arguments $A_1, \ldots, A_n$ is increasing whenever $n$ increases and larger than the maximum of the strengths of its parts $A_i$.

\[ f(x, y) = x + y - xy \]  \hspace{1cm} \text{Yanal (1991)} \hspace{1cm} (1)

\[ g(x, y) = 2x + 2y - 2xy - 1 \]  \hspace{1cm} \text{Selinger (2014)} \hspace{1cm} (2)

where in Selinger’s account $x, y > \frac{1}{2}$ to make the argument conclusive.

However, in Selinger’s approach a very bad additional argument (e.g. $y = 0.2$) decreases the overall strength of the argument such that it may fall below the threshold. For example, $g(0.6, 0.2) = 0.36$. \(\Rightarrow\) I consider this implausible.
(EWL) **Weakest Link Principle:** *The strength of a linked argument is the minimum strength of its required premises.*

An argument can be made that EWL is not convincing for linked arguments; they are, practically by definition, not really compatible with an exclusionary approach to argument strength.

(ALT) **Ampliative Linked Argument:** *The strength of a linked argument is equal to or above an upper threshold \( U \geq 0.5 \) for \( k \geq n \) premises and equal to or below a lower threshold \( L < 0.5 \) for \( k < n \) premises.*

This suggests a range of \( k \)-ary threshold functions \( S_n^k(x_1, \ldots, x_k) \) such that for \( k \geq n \), \( S_n^k(x_1, \ldots, x_k) \geq U \) and for \( k < n \), \( S_n^k(x_1, \ldots, x_k) \leq L \) close to zero. The functions can be increasing with increasing \( k \) (\( k \geq n \)) and decreasing with decreasing \( k \) (\( k < n \)).  

\( \Rightarrow \) Variants of Yanal’s Rule can be used in the function.
**Figure:** Four types of direct chains: (a) pro-pro, (b) pro-contra, (c) contra-pro, and (d) contra-contra chain.
**Argument Chain Strength (Exclusionary)**

**EWP) Weakest Pro Principle:** The strength of a pro-contra chain or a chain consisting of only pro links is the minimum of all strengths of the links in the chain.

**EGP) General Attack/Support Principle:** If $S(x_1, x_2, \ldots, x_n)$ is the function for the combined strength of a mixed chain $x_1, \ldots, x_n$, then $S$ is increasing in each pro argument $x_i$ and decreasing in each contra argument $x_j$.

- For pro chains, the exclusionary approach takes the minimum of the argument strengths.
**Contra Chain Strength** (Exclusionary)

(EC+) Contra Chain Maximum: The combined strength of a contra argument $C_1$ directly followed by a pro (contra) argument $C_2$ cannot be higher than the strength of $C_2$.

(EC-) Contra Chain Minimum: The combined strength of a contra argument $C_1$ directly followed by a pro (contra) argument $C_2$ is minimal whenever the strength of $C_1$ is maximal.

EGP, EC+, and EC- do not uniquely determine a function for contra-chains. However, in the absence of further criteria EC+ and EC- suggest

$$S(x, y) = y - xy$$  \hspace{1cm} (3)

➤ This is my proposal for contra chains.
(APC) Ampliative Pro Chain: The combined strength of a pro argument A directly followed by a pro or contra argument B goes against the minimum as the strength of A approaches the minimum, and against the strength of B as the strength of A approaches the maximum.

Pro chains: Multiplication satisfies APC and makes sense if each argument is independent of each other and if one treats the dependencies between arguments in a pro chain as a conjunction (t-norm in possibility theory).

Bipolar: Generally, bipolar ampliative approaches are underdeveloped. Cayrol (2009) give a general characterization but the two examples of evaluation functions they provide are not further motivated. I believe that EGP, EC+ and EC- are also acceptable for the ampliative approach.
**Link Attack/Support (Exclusionary)**

**(ELS) Link Support Principle:** The strength of an argument with a link supporter with initial strength $x$ for a link between $A$ and $b$ with strength $y$ is the minimum of $x$ and $y$.

- like EWP, the minimum principle for pro chains.

**(ELA) Link Attack Principle:** The strength of an argument with link attacker with initial strength $x$ for a link between $A$ and $b$ with strength $y$ is going against $y$ as $x$ goes against the minimum, and against the minimum as $x$ goes against the maximum.

- like EC+ and EC- for contra chains.
Link Attack/Support (Ampliative)

Two interpretations:

1. Support/attack like contra/pro argument chain, only that they pertain to a link instead of a conclusion.
   - Use the chain rule for attack:
     \[ g(x, y) = y - xy \]
   - Support like ampliative pro chain – i.e., it is treated like a conjunction.

2. Support/attack provide *additional* support or cast *additional* doubt.
   - Attack like above.
   - Support like ACU for bundles with Yanal’s Rule:
     \[ f(x, y) = x + y - xy \]
(EMB) Maximum Bundle Principle: *The strongest pro or contra argument uniquely determines the strength of a mixed bundle and multiple arguments in case of multiple argumentation; other arguments have no effect.*

(ERC) Reason Conflict Principle: *In a bundle a in which the strongest pro argument is as strong as the strongest contra argument, the strength of a is at the minimum.*

> While ERC looks reasonable, EMB exemplifies the limits of the exclusionary approach. However, being limited doesn’t mean that it’s wrong... (more on that later)
Mixed Bundles (Ampliative)

(ABP) Ampliative Bundle Principle: Bundles of the same polarity are treated like complex convergent arguments.

The theory may additionally be compensatory, which is reflected by the following principle:

(ACO) Compensatory Bundle Principle: The total strength of all pro bundles on C is weighed against the total strength of all contra bundles on C by a function that (i) is increasing in its first argument for the pro bundle and (ii) decreasing in its second argument for the contra bundle.
Mixed Bundles (Ampliative) Cont’d

Acceptable:

\[ S(x, y) = \text{Max}(0, x - y) \quad (4) \]
\[ S'(x, y) = x - xy \quad (5) \]

\( S \) is based on Łukasiewicz’s t-norm \( \text{Max}(0, x + y - 1) \), \( S' \) is less cautious.

Not acceptable:

\[ f(x, y) = -xy + 1 \quad (6) \]

because \( f(a, 0) = 1 \) even if \( a \) is close to 0.
Govier (2013): allow for *counterconsiderations*, also called *balance of consideration arguments*. That is, they are mixed pro/contra. However, I do not assume that they are special types of convergent argument. (This is just a terminological choice.)

- No exclusionary account reasonable – they are ampliative by definition.
- Difference to linked arguments: Strength does not fall to lower threshold whenever an essential premise is taken away.
- Difference to mixed bundles: Subarguments or premises of the same polarity do not first have to be combined, before weighing them against all arguments of the opposite polarity, or at least some additional justification would be needed to justify this particular mode of aggregation.
(ACA) Ampliative Conductive Argument: In a conductive argument the strengths of the pro premises $x_1, \ldots, x_k$ is weighed against the strengths of the contra premises $x_{k+1}, \ldots, x_n$ by a function $F(x_1, \ldots, x_k; x_{k+1}, \ldots, x_n)$ that is increasing in $x_1, \ldots, x_k$ and decreasing in $x_{k+1}, \ldots, x_n$. If $p$ is pro (contra), then $F$ approaches its maximum as $x_1, \ldots, x_k$ approach their maximum (minimum) values and $x_{k+1}, \ldots, x_n$ approach their minimum (maximum) values, and $F$ approaches its minimum as $x_1, \ldots, x_k$ approach their minimum (maximum) values and $x_{k+1}, \ldots, x_n$ approach their maximum (minimum) values.

Let $w_i = 1$ if $p_i = \oplus$, $w_i = -1$ otherwise. Then the normalized sum fulfills ACA:

$$F(x_1, \ldots, x_n) = \frac{1}{n} \sum_{i=1}^{n} w_i x_i \quad (7)$$

⚠️ But there is a huge problem!
Conductive arguments seem to incorrectly mix up argument strength with the *evaluative contribution* of reasons.

1. An argument for and support for a certain level of a cardinal or ordinal value attribute such as ‘quality of life’ or ‘costs’ can be more or less strong.

2. There may be different types of value or criteria both formally (e.g. additive vs. lexicographic) and substantially (e.g. different qualities of value in value pluralism) but there is only one degree of argument strength.

3. It is known from multiattribute decision making that numerical representations of multiple criteria that are ordered by preferences must satisfy corresponding representation theorems.
The Example (on the whiteboard, if there is interest)

Figure: Example of a BAG with 5 evaluation steps $\alpha - \epsilon$. 
Does Argument Strength Exist?

• Why is there no uniform notion of argument strength?
• We have seen conflicting intuitions, but there are deeper reasons.
• Strength can pertain either to:
  • the premises \( \rightsquigarrow (A) \) adequacy of evidence / support
  • the inferences \( \rightsquigarrow (B) \) fallacies and (C) probability that inference is valid/correct
• (A) and (C) can only be addressed by a fully-fledged formal epistemology, if a fully justified measure of strength is desired.
• However, such a theory is (i) philosophically dubious (e.g. no ‘logic of discovery’), (ii) would not be practical (highly idealized), and (iii) no longer reflects intuitive argument strength.

⇒ The principles I've discussed can only ever remain a rough approximation. Add any more detail and you end up doing formal epistemology within a particular framework (probabilistic, possibilistic, etc.)
Conclusions

- Psychological argument strength is not argument strength – it deals with *perceived* argument strength.
- Argument strength might after all be a convenient fiction.
- But it may also be considered a rough approximation of justifiable estimates.
- Better theories need to take into account the inference type (deductive, inductive, maybe also abductive) and probabilistically sound descriptions of premises, evidence, and sources of error.
References